Multimedia Indexing and Retrieval

Deep Learning for multimedia indexing and retrieval

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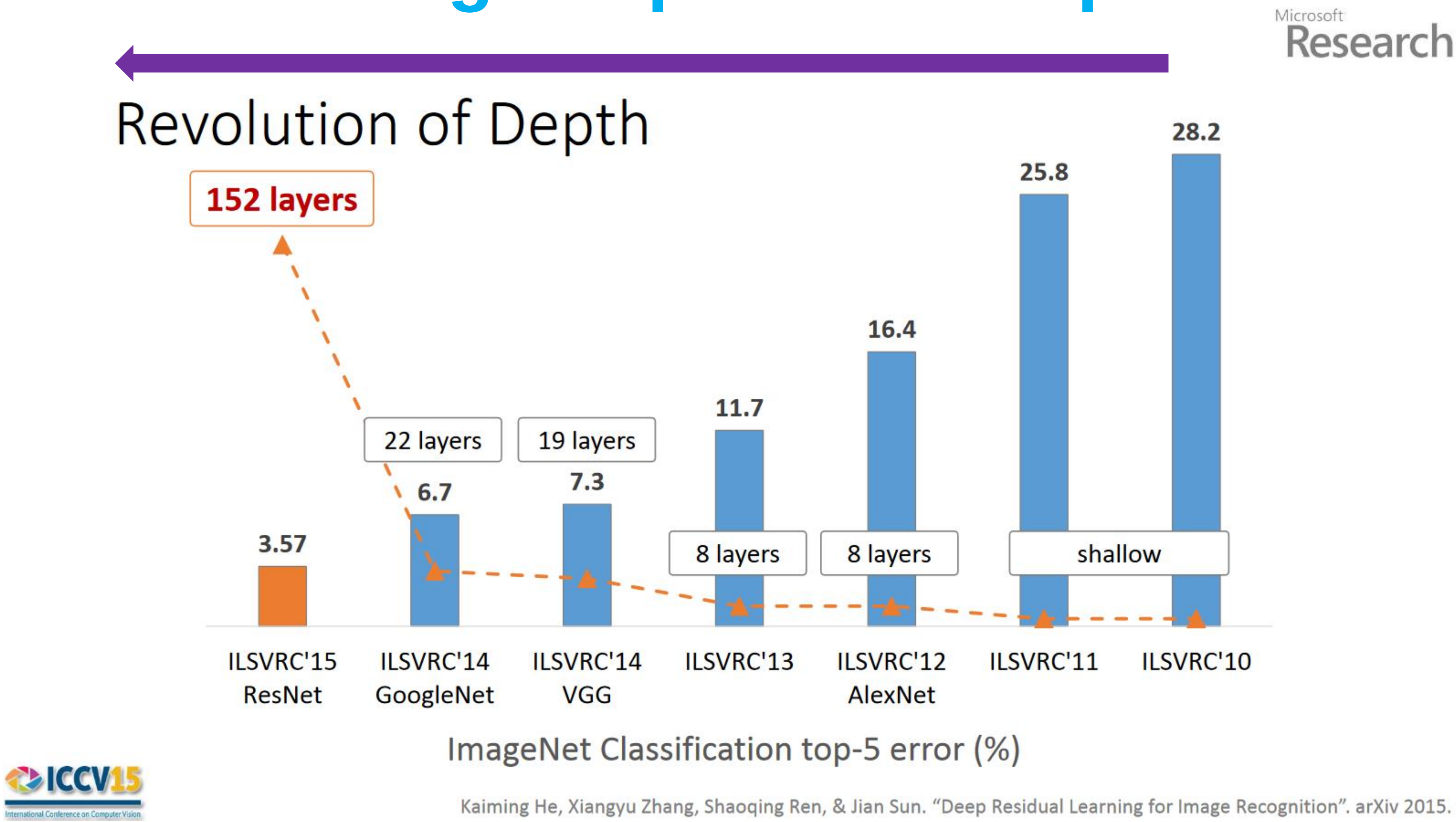
Outline

- Introduction
- Machine learning
- Loss function
- Formal neuron
- Single layer perceptron
- Multilayer perceptron
- Reminders about differential calculus
- Back-propagation
- Learning rate
- Mini-batches
- Convolutional layers
- Pooling, softmax …

ImageNet Large Scale Visual Recognition Challenge (ILSVRC)

- 1000 visual “fine grain” categories / labels (exclusive)
- 150,000 test images (hidden “ground truth”)
- 50,000 validation images
- 1,200,000 training images
- Each training, validation or test image falls within exactly one of the 1000 categories
- Task: for each image in the test set, rank the categories from most probable to least probable
- Metric: top-5 error rate: percentage of images for which the actual category is not in the five first ranked categories
- Held from 2010 to 2015, frozen since 2012

Going deeper and deeper



For comparison, human performance is 5.1% (Russakovsky et al.)

Deep Convolutional Neural Networks

- Decades of algorithmic improvements in neural networks (Stochastic Gradient Descent, initialization, momentum …)
- Very large amounts of properly annotated data (ImageNet)
- Huge computing power (Teraflops × weeks): GPU!
- Convolutional networks
- Deep networks ($>>$ 3 layers)
- ReLU (Rectified Linear Unit) activation functions
- Batch normalization
- Drop Out
- …

Deep Learning is (now) EASY

- Maths: linear algebra and differential calculus (training only)
  - $Y = A.X + B$ (with tensor extension)
  - $f(x + h) = f(x) + f'(x).h + o(h)$ (with multidimensional variables)
  - $(g\ o\ f)'(x) = (g'o\ f)(x).f'(x)$ (recursively applied)
- Tools: amazingly integrated, effective and easy to use packages
  - Mostly python interface
  - Autograd packages: only need to care of the linear algebra part
- Get started with:
  - 3-hour course
  - 1-hour PyTorch tutorial (familiarity with python assumed)

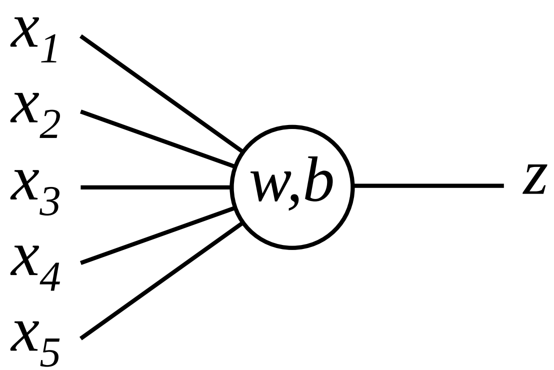
Parametric supervised learning

- Parameterized function: $f : \mathbb{R}^m \rightarrow Y^X$
  $\theta \rightarrow f_\theta$
- f is a "meta" function or a family of function
- Target function: $f_\theta : X \rightarrow Y$
  $x \rightarrow y = f_\theta(x)$
  - X : set of valid input objects ($\mathbb{R}^d$)
  - Y : set of possible output values ($\mathbb{R}^c$)
- Training data: $S = (x_i, y_i)_{(1 \leq i \leq I)}$
  - I : number of training samples
- Learning algorithm: $L_f : (X \times Y)^* \rightarrow \mathbb{R}^m$ (learns θ from S)
  $S \rightarrow \theta = L_f(S)$
- Regression or classification system: $y = f_\theta(x) = f(\theta, x)$

Multi-label loss function

- Predict P labels for each data sample x
- P decision functions : $f = (f_p)_{(1 \leq p \leq P)}$
- Example with f depending on a parameter vector:
  $E_S(\theta) = \frac{1}{2}\sum_{i=1}^{i=I}\sum_{p=1}^{p=P}(f_p(\theta, x_i) - y_{ip})^2 = \frac{1}{2}\sum_{i=1}^{i=I}(f(\theta, x_i) - y_i)^2$
  (same as single label case with Euclidean distance between vectors of predictions and vectors of labels)
- $\theta^* = \underset{\theta}{\operatorname{argmin}}\ E_S(\theta)$
- The f_p functions may take any real value

Formal neural or unit (two sub-units)



linear and vector part

$$y = \sum_j w_j x_j = w.x$$

linear combination

x : column vector
w : row vector

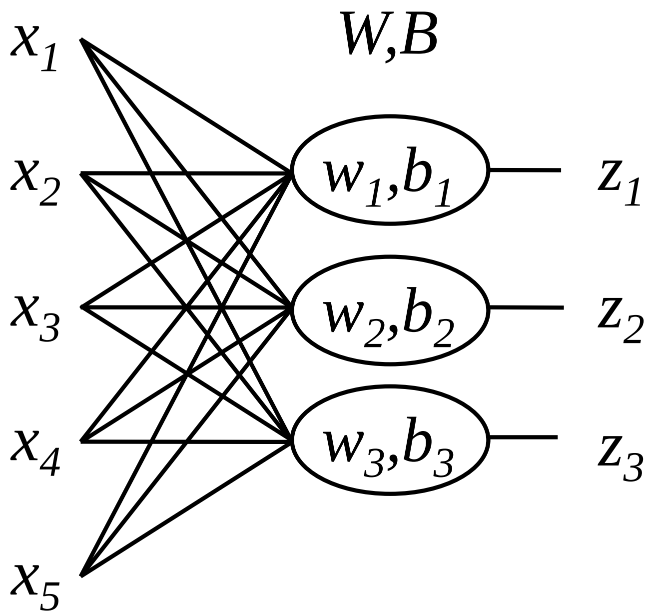
non-linear and scalar part

$$z = \sigma(y + b) = \frac{1}{1 + e^{y+b}}$$

sigmoid function

y, b, z : scalars

Neural layer (all to all, two sub-layers)



$$y_i = \sum_j w_{ij} x_j$$

matrix-vector multiplication

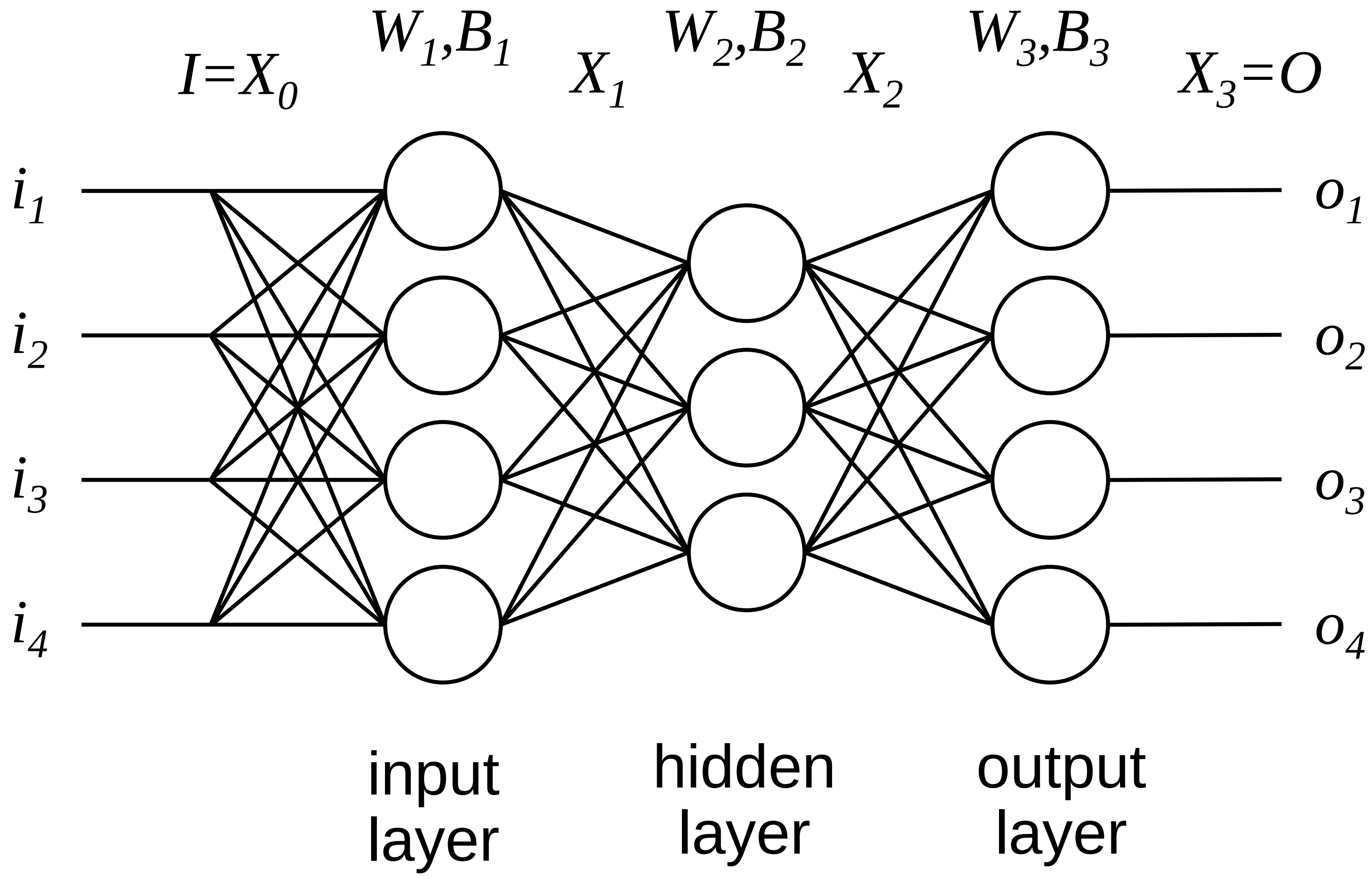
$$Y = W.X$$

$$z_i = \sigma(y_i + b_i) = \frac{1}{1 + e^{y_i + b_i}}$$

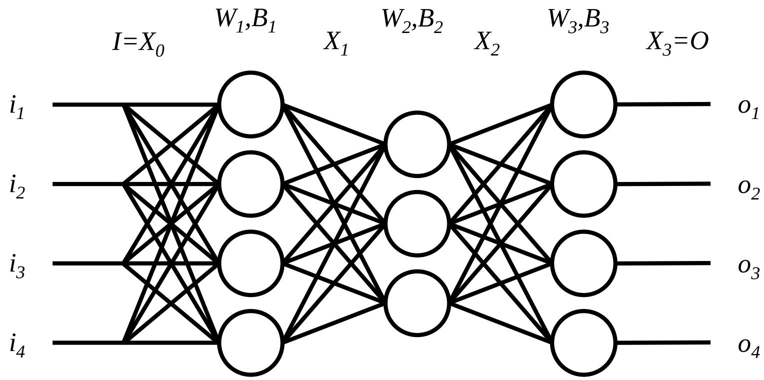
per component operation

$$z = \sigma(Y + B)$$

Multilayer perceptron (all to all)



Multilayer perceptron (all to all)



$$Y_1 = W_1.X_0 = F_1(W_1, X_0) \qquad X_1 = \sigma(Y_1 + B_1) = G_1(B_1, Y_1)$$
$$Y_2 = W_2.X_1 = F_2(W_2, X_1) \qquad X_2 = \sigma(Y_2 + B_2) = G_2(B_2, Y_2)$$
$$Y_3 = W_3.X_3 = F_3(W_3, X_2) \qquad X_3 = \sigma(Y_3 + B_3) = G_3(B_3, Y_3)$$

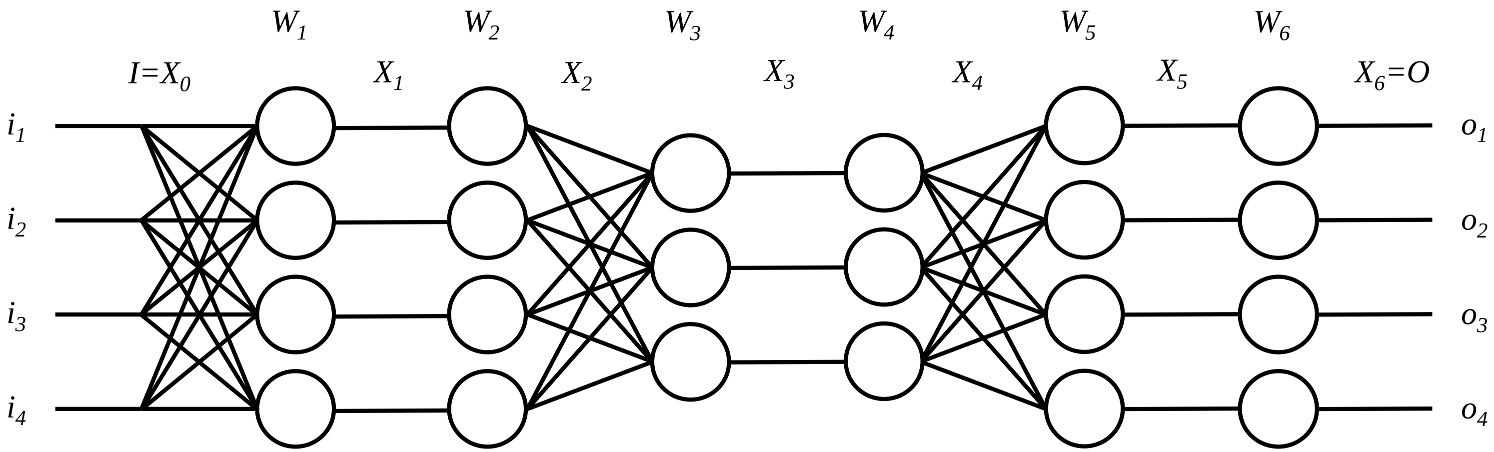
$$O = X_3 = G_3\left(B_3, F_3\left(W_3, G_2\left(B_2, F_2\left(W_2, G_1\left(B_1, F_1(W_1, X_0 = I)\right)\right)\right)\right)\right)$$

Denoting $F(W)$ so that $F(W, X) = (F(W))(X)$:

$$O = \left(G_3(B_3)\ o\ F_3(W_3)\ o\ G_2(B_2)\ o\ F_2(W_2)\ o\ G_1(B_1)\ o\ F_1(W_1)\right)(I)$$

Composition of simple functions

Splitting units and layers, renaming and renumbering:



$$X_1 = W_1.X_0 = F_1(W_1, X_0) \qquad X_2 = \sigma(X_1 + W_2) = F_2(W_2, X_1)$$

$$X_3 = W_3.X_2 = F_3(W_3, X_2) \qquad X_4 = \sigma(X_3 + W_4) = F_4(W_4, X_3)$$

$$X_5 = W_5.X_4 = F_5(W_5, X_4) \qquad X_6 = \sigma(X_5 + W_6) = F_6(W_6, X_5)$$

$$O = \left(F_6(W_6)\ o\ F_5(W_5)\ o\ F_4(W_4)\ o\ F_3(W_3)\ o\ F_2(W_2)\ o\ F_1(W_1)\right)(I) = \left(o_{n=1}^{n=6}\ F_n(W_n)\right)(I)$$

Composition of simple functions

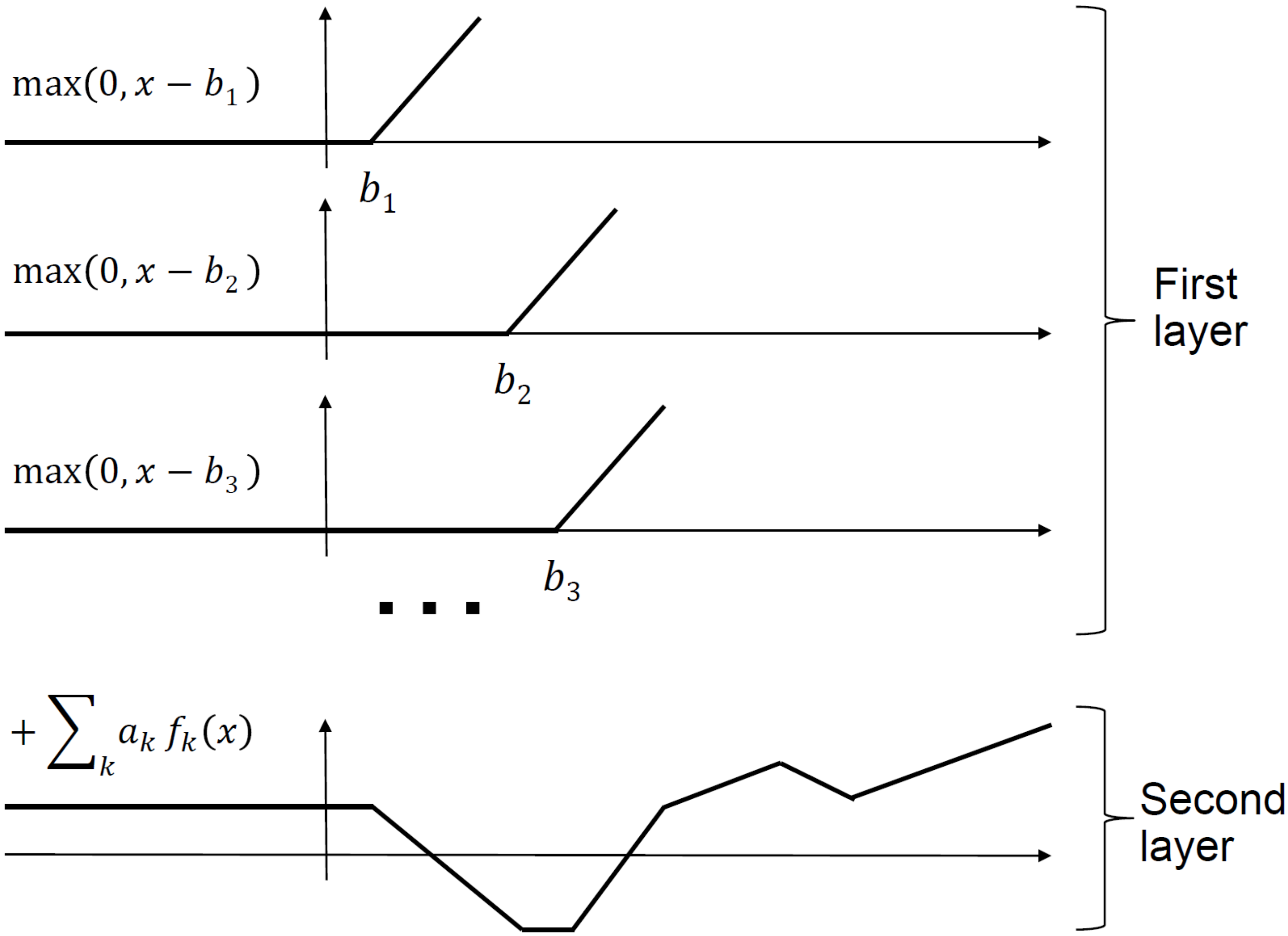
$f_1(x) = \max(0, x - b_1)$

$f_2(x) = \max(0, x - b_2)$

$f_3(x) = \max(0, x - b_3)$

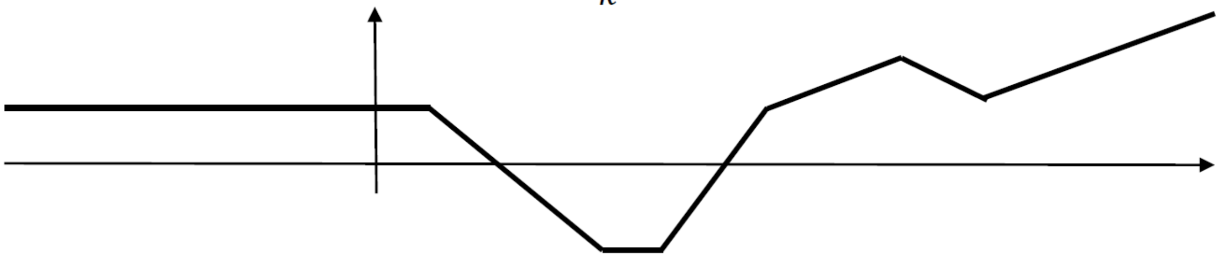
…

$f(x) = a_0 + \sum_k a_k f_k(x)$



Composition of simple functions

$$f_\theta(x) = a_0 + \sum_k a_k \max(0, x - b_k)$$



- Model parameters: $\theta = (a_0, a_1, b_1, a_2, b_2\ \ldots)$
- Empirical risk on training data: $E(\theta) = \sum_i \left(y_i - f_\theta(x_i)\right)^2$
- Find the optimal function by gradient descent on θ
- Any function can do: sigmoids, gaussians, sin/cos …
- ReLU is simpler and converges faster
- More layers: more complex functions with less parameters

Feed Forward Network

- Global network definition: $O = F(W, I)$
  ($I \equiv x \;\; O \equiv y \;\; F \equiv f \;\; W \equiv \theta$ relative to previous notations)

- Layer values: $(X_0, X_1 \text{ ... } X_N)$
  with $X_0 = I$ and $X_N = O$ (X_n are vectors)

- Global vector of all unit parameters:
  $W = (W_1, W_2 \text{ ... } W_N)$
  (weights by layer are concatenated, W_n can matrices or vectors or any parameter structure, and even possibly empty)

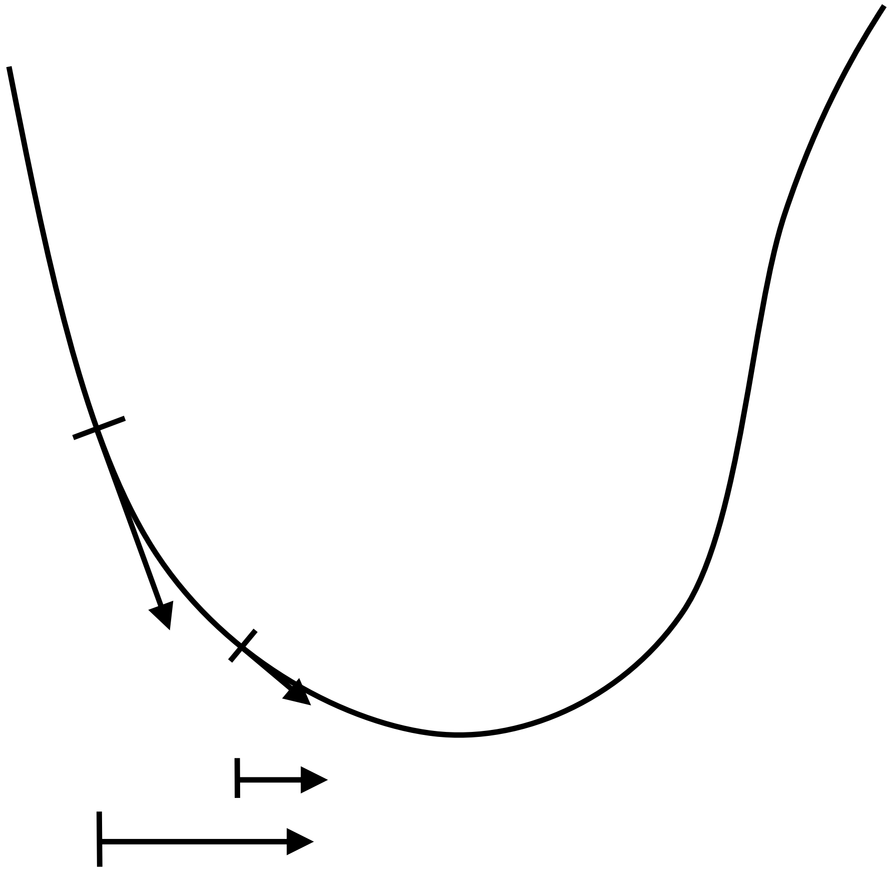
- Feed forward: $X_{n+1} = F_{n+1}(W_{n+1}, X_n)$

- Possibly "joins" and "forks" (but no cycles)

Error back-propagation

- Training set: $S = (I_i, O_i)_{(1 \leq i \leq I)}$ input-output samples
- $X_{i,0} = I_i$ and $X_{i,n+1} = F_{n+1}\left(W_{n+1}, X_{i,n}\right)$
- Note: regarding this notation the vector-matrix multiplication counts as one layer and the element-wise non-linearity counts as another one (not mandatory but greatly simplifies the layer modules' implementation)
- Error (empirical risk) on the training set:
  $E(W) = \sum_i (F(W, I_i) - O_i)^2 = \sum_i \left(X_{i,N} - O_i\right)^2$
- Minimization of $E(W)$ by gradient descent

Gradient descent



Error back-propagation

- Minimization of $E_S(W)$ by gradient descent:
  - The gradient indicate an ascending direction: move in the opposite
  - Randomly initialize $W(0)$
  - Iterate $W(t+1) = W(t) - \eta \frac{\partial E}{\partial W}\big(W(t)\big)$ $\quad \eta = f(t)$ or $\left(\frac{\partial^2 E}{\partial W^2}\big(W(t)\big)\right)^{-1}$
  - $\frac{\partial E}{\partial W} = \left(\frac{\partial E}{\partial W_1}, \frac{\partial E}{\partial W_2} \ \dots \ \frac{\partial E}{\partial W_N}\right)$ $\qquad (W = (W_1, W_2 \ \dots \ W_N))$
  - Back-propagation: $\frac{\partial E}{\partial W_n}$ is computed by backward recurrence from $\frac{\partial F_n}{\partial W_n}$ and $\frac{\partial F_n}{\partial X_{n-1}}$ applying iteratively $(g \ o \ f)' = (g' o \ f).f'$
  - Two derivatives, relative to weight and to data to be considered

Differential of a function
scalar input and scalar output

- $f : \mathbb{R} \to \mathbb{R} : x \to f(x)$ $\qquad$ f is differentiable
- $y = f(x)$
- $f(x) - f(x + h) = f'(x)h + o(h) \quad (\lim_{h \to 0} \frac{o(h)}{h} = 0)$
- $dy = f'(x)dx$ $\qquad$ i.e: f is “locally linear”
- $\frac{dy}{dx} \equiv f'(x)$ $\qquad$ (notation)
- $dy = \frac{dy}{dx} dx$ $\qquad$ (“local scale factor”)
- All values are scalar

Differential of a composed function scalar input and scalar output

- $f : \mathbb{R} \to \mathbb{R} : x \to f(x)$ f is differentiable
- $y = f(x)$
- $g : \mathbb{R} \to \mathbb{R} : y \to g(y)$ g is differentiable
- $z = g(y)$
- $(g\ o\ f)'(x) = (g'o\ f)(x).f'(x) = g'(y).f'(x)$
- $dy = \frac{dy}{dx} dx$ $dz = \frac{dz}{dy} dy$
- $dz = \frac{dz}{dy} \cdot \frac{dy}{dx} dx$ $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

Differential of a function of a vector vector input and scalar output

- $f : \mathbb{R}^N \rightarrow \mathbb{R} : x \rightarrow f(x)$ $\quad$ f is differentiable
- $y = f(x)$ $\quad$ $x = (x_i)_{(1 \leq i \leq N)}$
- $f(x) - f(x+h) = \text{grad}\, f\,(x).h + o(\|h\|)$
- $dy = \text{grad}\, f\,(x).dx = \sum_{i=1}^{i=n} \frac{\partial f}{\partial x_i}(x).dx_i = \sum_{i=1}^{i=n} \frac{\partial y}{\partial x_i}.dx_i = \frac{\partial y}{\partial x}.dx$
- $\frac{\partial y}{\partial x} \equiv \frac{\partial f}{\partial x}(x) = \text{grad}\, f\,(x)$ $\quad$ $\frac{\partial y}{\partial x_i} \equiv \frac{\partial f}{\partial x_i}(x)$ $\quad$ (notations)
- y, dy and $f(x)$ are scalars;
- x, dx and h are "regular" (column) vectors;
- $\frac{\partial y}{\partial x}$ is a transpose (row) vector.

Differential of a vector function of a vector vector input and vector output

- $f : \mathbb{R}^N \rightarrow \mathbb{R}^P : x \rightarrow f(x)$ f is differentiable
- $y = f(x)$ $x = (x_i)_{(1 \leq i \leq N)}$ $y = (y_j)_{(1 \leq j \leq P)}$ $f = (f_j)_{(1 \leq j \leq P)}$
- $f(x) - f(x+h) = \text{grad}\, f\,(x).h + o(\|h\|)$
- $dy = \text{grad}\, f\,(x).dx = \frac{\partial f}{\partial x}(x).dx = \frac{\partial y}{\partial x}.dx$ (locally linear)
- $dy_j = \sum_{i=1}^{i=n} \frac{\partial f_j}{\partial x_i}(x).dx_i = \sum_{i=1}^{i=n} \frac{\partial y_j}{\partial x_i}.dx_i$
- x, dx, y, dy, $f(x)$ and h are all "regular" vectors;
- $\frac{\partial y}{\partial x}$ is a matrix (Jacobian of f: $J_{ij} = \left(\frac{\partial y}{\partial x}\right)_{ij} = \frac{\partial y_j}{\partial x_i} = \frac{\partial f_j}{\partial x_i}(x)$).

Differential of a composed function vector inputs and vector outputs

- $f : \mathbb{R}^N \to \mathbb{R}^P : x \to y = f(x)$ $\quad$ f is differentiable
- $g : \mathbb{R}^P \to \mathbb{R}^Q : y \to z = g(y)$ $\quad$ g is differentiable
- $x = (x_i)_{(1 \leq i \leq N)}$ $\quad$ $y = (y_j)_{(1 \leq j \leq P)}$ $\quad$ $z = (z_k)_{(1 \leq k \leq Q)}$
- $dz = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x} . dx$
- $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$ $\quad$ (matrix multiplication: non commutative!)
- x, dx, y, dy, z, dz, $f(x)$ and $g(y)$ are all regular vectors;
- $\frac{\partial y}{\partial x}$, $\frac{\partial z}{\partial y}$ and $\frac{\partial z}{\partial x}$ are all matrices (f, g and gof Jacobians).

Differential of a composed function vector inputs and scalar output

- $f : \mathbb{R}^N \to \mathbb{R}^P : x \to y = f(x)$ $\quad$ f is differentiable
- $g : \mathbb{R}^P \to \mathbb{R} \quad : y \to z = g(y)$ $\quad$ g is differentiable
- $x = (x_i)_{(1 \leq i \leq N)}$ $\quad$ $y = (y_j)_{(1 \leq j \leq P)}$ $\quad$ $z \in \mathbb{R}$
- $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$ $\quad$ (left row vector × matrix mult. → row vector)
- z, dz and $g(y)$ are scalars;
- x, dx, y, dy, and $f(x)$ are regular vectors;
- $\frac{\partial z}{\partial y}$ and $\frac{\partial z}{\partial x}$ are transpose (row) vectors (f and gof gradients);
- $\frac{\partial y}{\partial x}$ is a matrix (f Jacobian).

Error back-propagation

- Minimization of $E_S(W)$ by gradient descent:
  - The gradient indicate an ascending direction: move in the opposite
  - Randomly initialize $W(0)$
  - Iterate $W(t+1) = W(t) - \eta \frac{\partial E}{\partial W}(W(t))$ $\eta = f(t)$ or $\left(\frac{\partial^2 E}{\partial W^2}(W(t))\right)^{-1}$
  - $\frac{\partial E}{\partial W} = \left(\frac{\partial E}{\partial W_1}, \frac{\partial E}{\partial W_2} \ \dots \ \frac{\partial E}{\partial W_N}\right)$ $\quad (W = (W_1, W_2 \ \dots \ W_N))$
  - Back-propagation: $\frac{\partial E}{\partial W_n}$ is computed by backward recurrence from $\frac{\partial F_n}{\partial W_n}$ and $\frac{\partial F_n}{\partial X_{n-1}}$ applying iteratively $(g \ o \ f)' = (g' o \ f).f'$
  - Two derivatives, relative to weight and to data to be considered

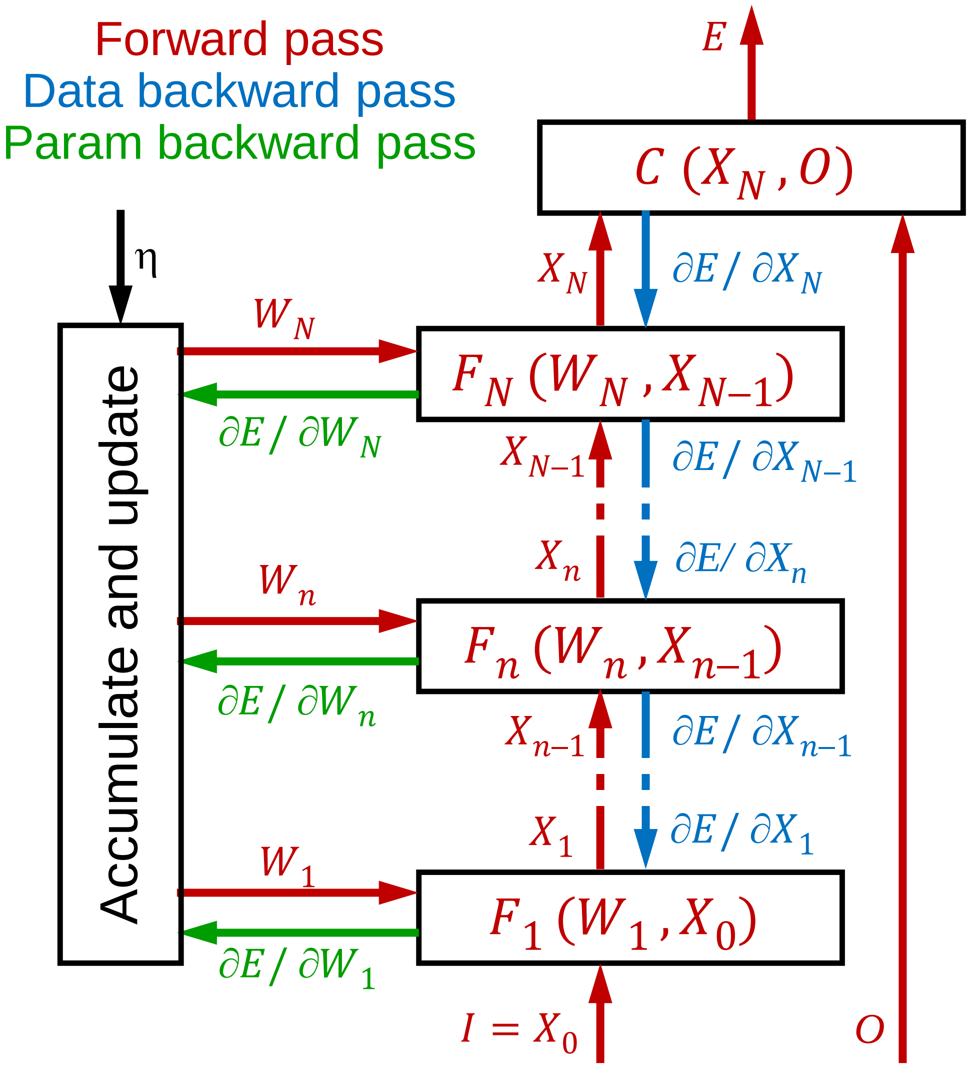
Stochastic gradient descent and batch processing

- $E(W) = \sum_i (F(W, I_i) - O_i)^2 = \sum_i E_i(W)$
- $W(t+1) = W(t) - \eta(t)\frac{\partial E}{\partial W}(t) = W(t) - \sum_i \eta(t)\frac{\partial E_i}{\partial W}(t)$
- Global update (epoch): sum of per sample updates
- Classical GD: update W globally after all I samples have been processed ($1 \leq i \leq I$)
- Stochastic GD: update W after each processed sample $\rightarrow$ immediate effect, faster convergence
- Batch: update W after a given number (typically between 32 and 256) of processed samples $\rightarrow$ parallelism

Learning rate evolution

- $W(t+1) = W(t) - \eta(t)\frac{\partial E}{\partial W}\big(W(t)\big)$
- Large learning rate: instability
- Small learning rate: very slow convergence
- Variable learning rate: learning rate decay policy
- Most often: step strategy: iterate “constant during a number of epochs, then divide by a given factor”
- Possibly different learning rates for different layers or for different types of parameters, generally with common evolution

Error back-propagation (adapted from Yann LeCun)



Forward pass, for $1 \leq n \leq N$:

$X_n = F_n\ (W_n, X_{n-1})$

$E = C\ (X_N, O)$

We need gradients with respect to X_n. For N:

$$\frac{\partial E}{\partial X_N} = \frac{\partial C(X_N, O)}{\partial X_N}$$

Then backward recurrence:

$$\frac{\partial E}{\partial X_{n-1}} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(W_n, X_{n-1})}{\partial X_{n-1}}$$

Gradients with respect to W_n.
For $1 \leq n \leq N$:

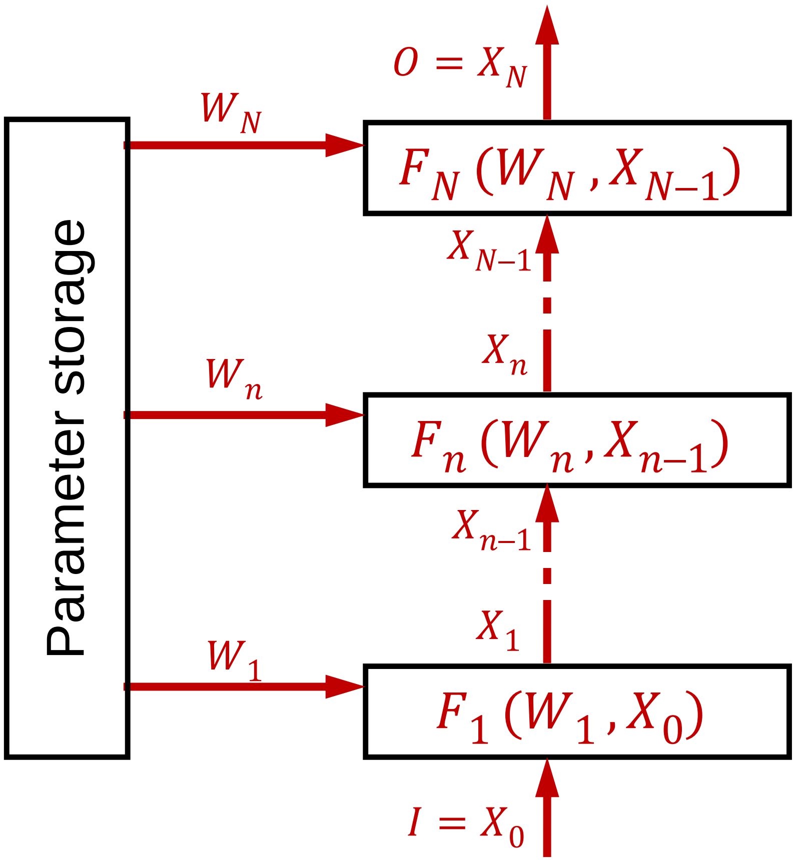
$$\frac{\partial E}{\partial W_n} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(W_n, X_{n-1})}{\partial W_n}$$

Error back-propagation 0: Prediction mode

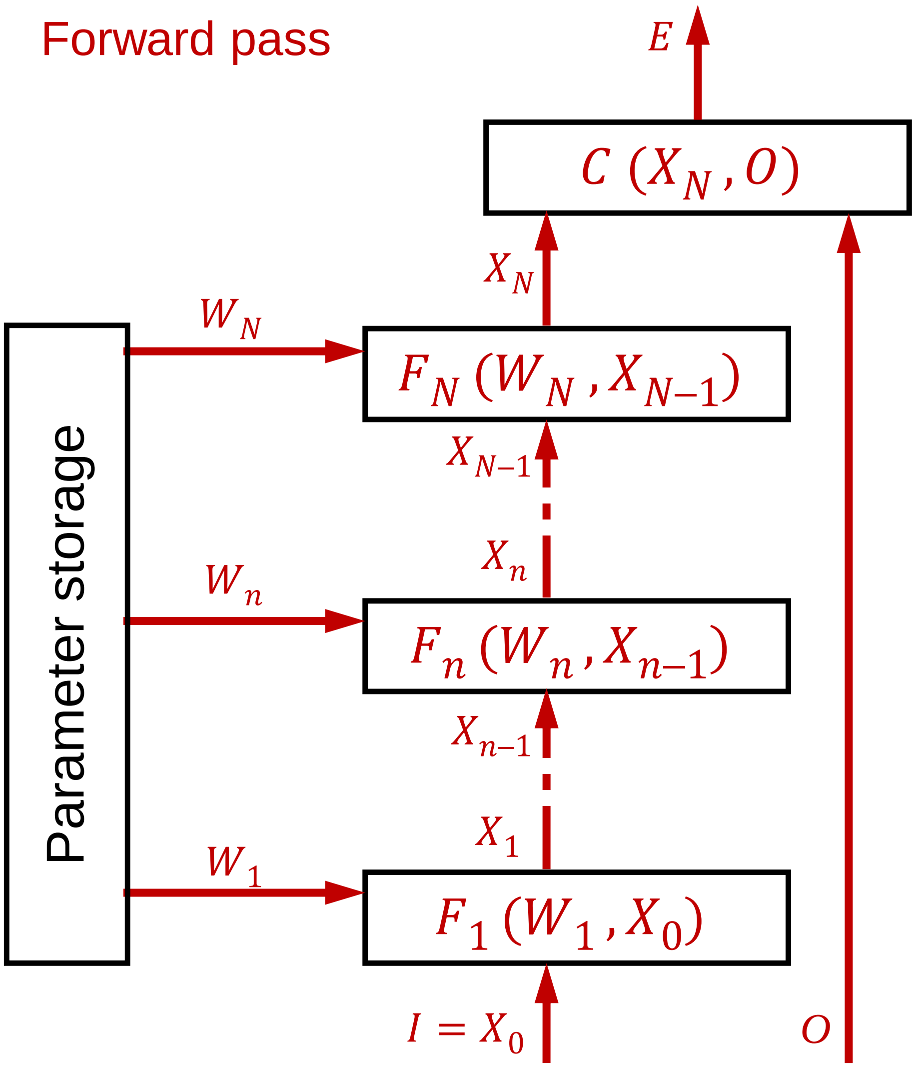
Forward pass

Forward pass, for $1 \leq n \leq N$:

$X_n = F_n(W_n, X_{n-1})$



Error back-propagation 1: loss function



Forward pass, for $1 \leq n \leq N$:

$X_n = F_n\,(W_n\,,X_{n-1})$

Loss function (for one sample):

$E = C\,(X_N\,,O)$

$E(W,I,O) = C(F(W,I),O)$

Sum over the whole training set or over a batch of samples:

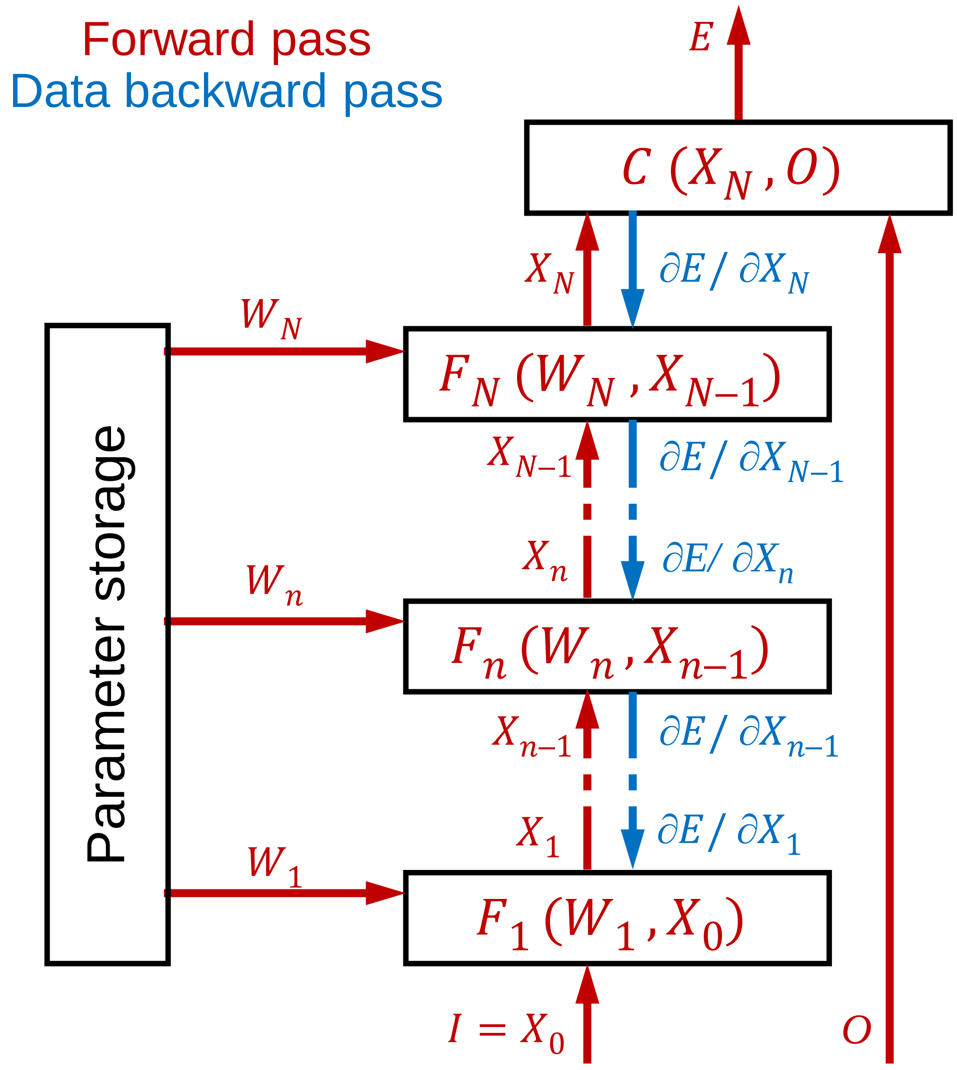
$$E(W) = \sum_i E(W,I_i,O_i)$$

Same W, different (I_i,O_i)

Update:

$$W = W - \eta\frac{\partial E(W)}{\partial W}$$

Error back-propagation 2: Data backward pass



Forward pass, for $1 \leq n \leq N$:

$X_n = F_n\ (W_n, X_{n-1})$

$E = C\ (X_N, O)$

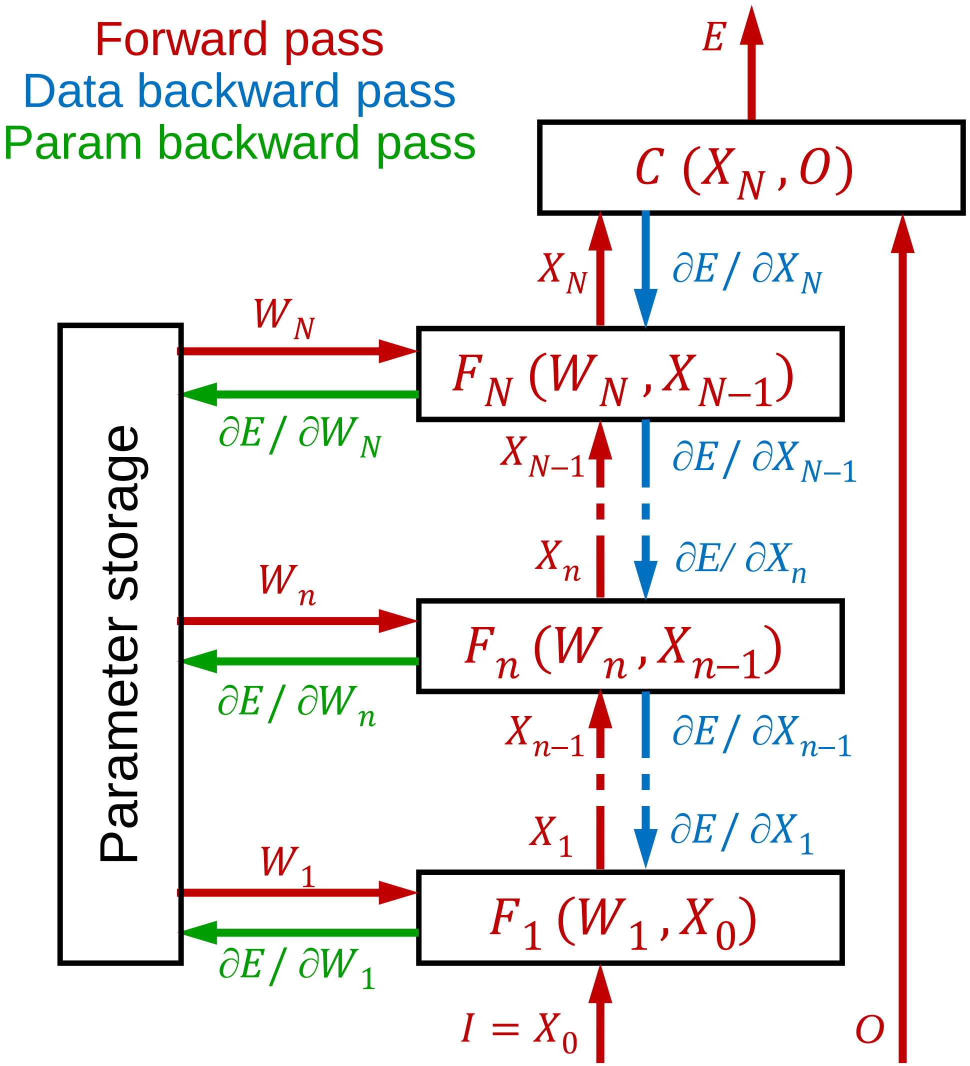
We need gradients with respect to X_n. For N:

$$\frac{\partial E}{\partial X_N} = \frac{\partial C(X_N, O)}{\partial X_N}$$

Then backward recurrence:

$$\frac{\partial E}{\partial X_{n-1}} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(W_n, X_{n-1})}{\partial X_{n-1}}$$

Error back-propagation 3: Parameter backward pass



Forward pass, for $1 \leq n \leq N$:

$X_n = F_n(W_n, X_{n-1})$

$E = C(X_N, O)$

We need gradients with respect to X_n. For N:

$$\frac{\partial E}{\partial X_N} = \frac{\partial C(X_N, O)}{\partial X_N}$$

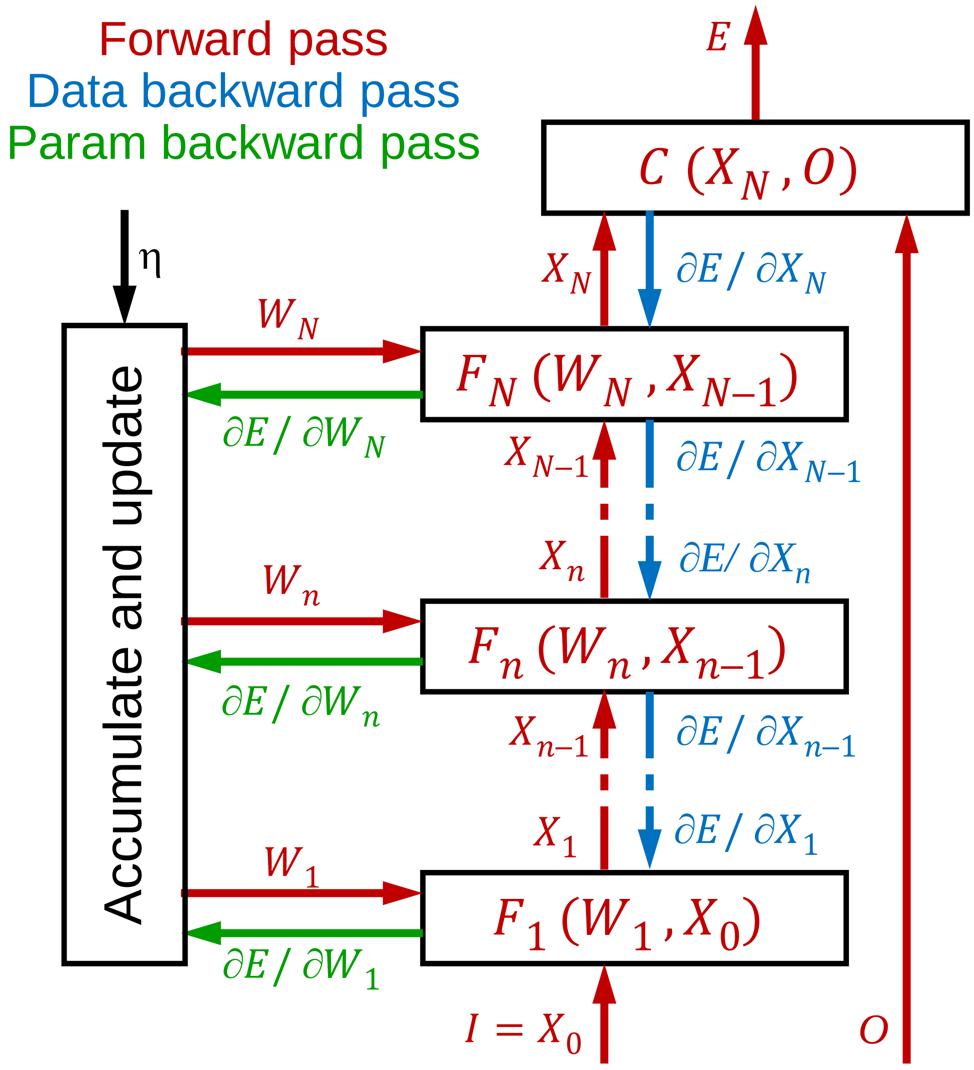
Then backward recurrence:

$$\frac{\partial E}{\partial X_{n-1}} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(W_n, X_{n-1})}{\partial X_{n-1}}$$

Gradients with respect to W_n. For $1 \leq n \leq N$:

$$\frac{\partial E}{\partial W_n} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(W_n, X_{n-1})}{\partial W_n}$$

Error back-propagation 4: Accumulate and update



Forward pass, for $1 \leq n \leq N$:

$X_n = F_n\ (W_n, X_{n-1})$

$E = C\ (X_N, O)$

…

Gradients with respect to W_n.

For $1 \leq n \leq N$:

$$\frac{\partial E}{\partial W_n} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(W_n, X_{n-1})}{\partial W_n}$$

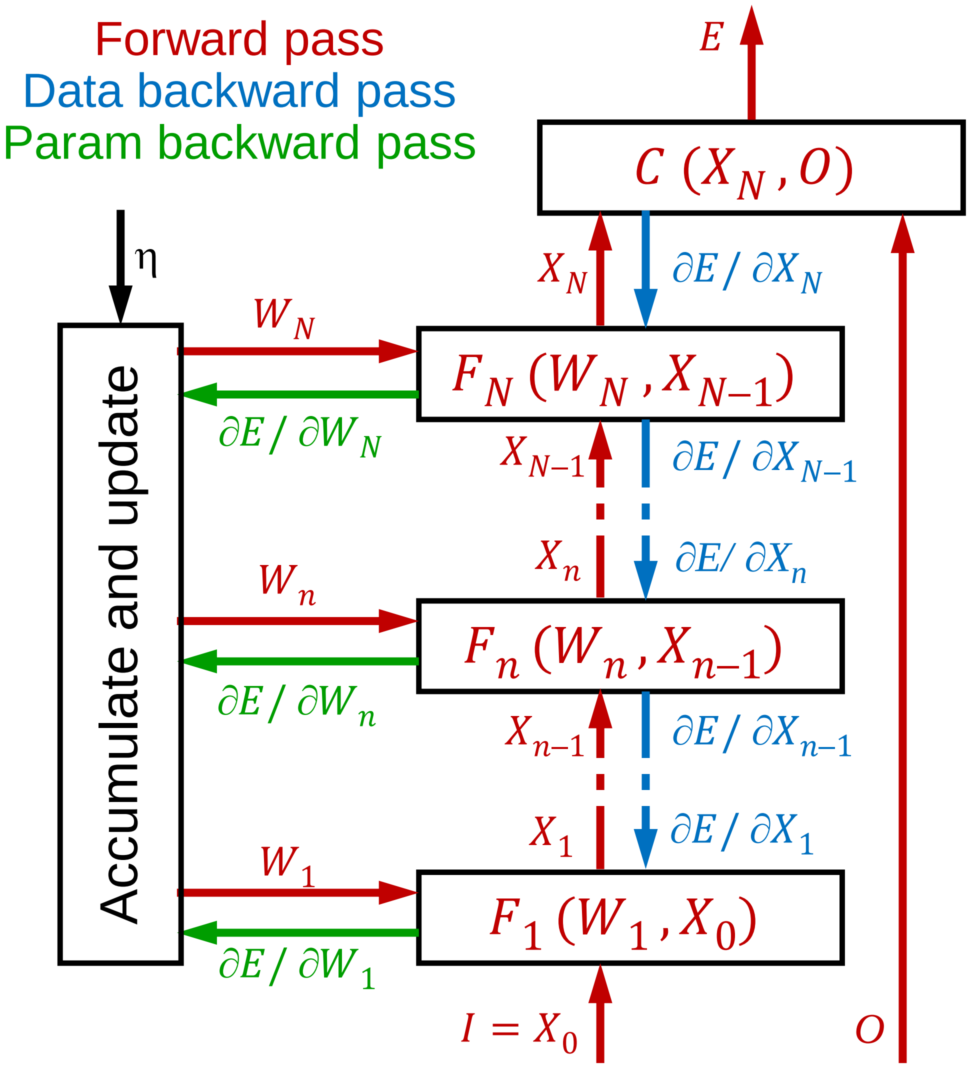
Accumulate gradients and update parameters.

For $1 \leq n \leq N$:

$$W_n = W_n - \eta \sum_i \frac{\partial E}{\partial W_n}(W, I_i, O_i)$$

Usually on batches

Error back-propagation: simplified notations



Forward pass, for $1 \leq n \leq N$:

$X_n = F_n(W_n, X_{n-1})$

$E = C(X_N, O)$

We need gradients with respect to X_n. For N:

$$\frac{\partial E}{\partial X_N} = \frac{\partial C}{\partial X_N}$$

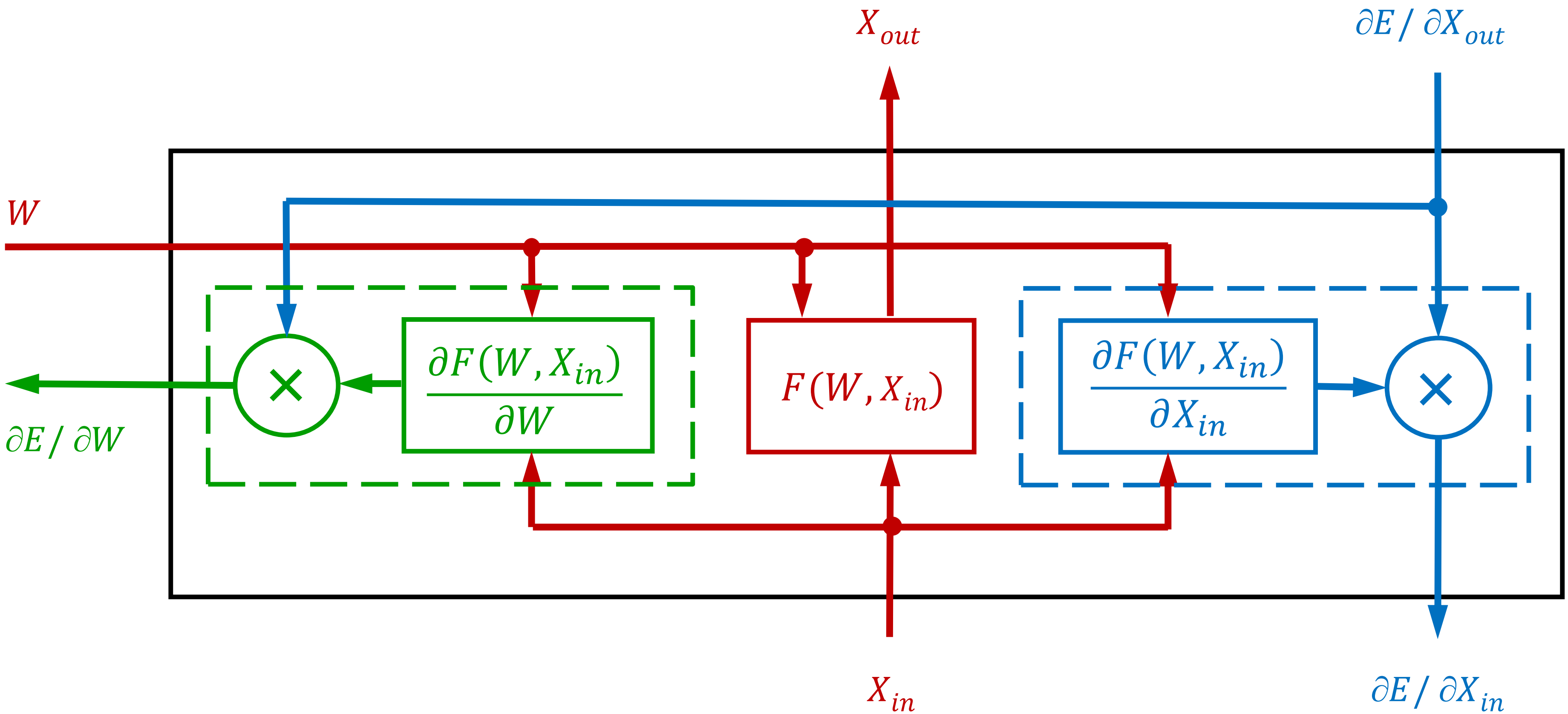
Then backward recurrence:

$$\frac{\partial E}{\partial X_{n-1}} = \frac{\partial E}{\partial X_n}\frac{\partial X_n}{\partial X_{n-1}}$$

Gradients with respect to W_n. For $1 \leq n \leq N$:

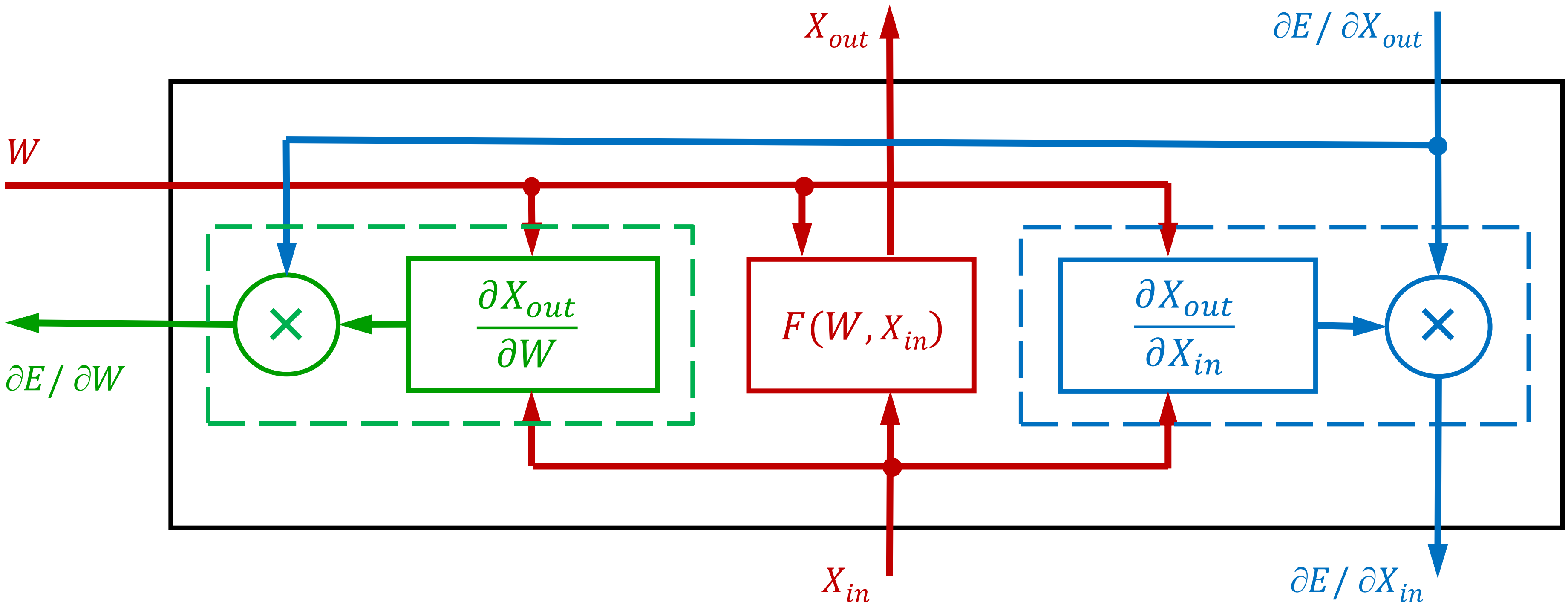
$$\frac{\partial E}{\partial W_n} = \frac{\partial E}{\partial X_n}\frac{\partial X_n}{\partial W_n}$$

Layer module (adapted from Yann LeCun)



Notes: $X_{in} \equiv X_{n-1}$, $X_{out} \equiv X_n$, $W \equiv W_n$ and $F \equiv F_n$ for $1 \leq n \leq N$

Layer module (adapted from Yann LeCun)



$$\frac{\partial F(W, X_{in})}{\partial X_{in}} \equiv \frac{\partial X_{out}}{\partial X_{in}} \qquad \frac{\partial E}{\partial X_{in}} = \frac{\partial E}{\partial X_{out}} \frac{\partial X_{out}}{\partial X_{in}}$$

$$\frac{\partial F(W, X_{in})}{\partial W} \equiv \frac{\partial X_{out}}{\partial W} \qquad \frac{\partial E}{\partial W} = \frac{\partial E}{\partial X_{out}} \frac{\partial X_{out}}{\partial W}$$

Layer module (adapted from Yann LeCun)

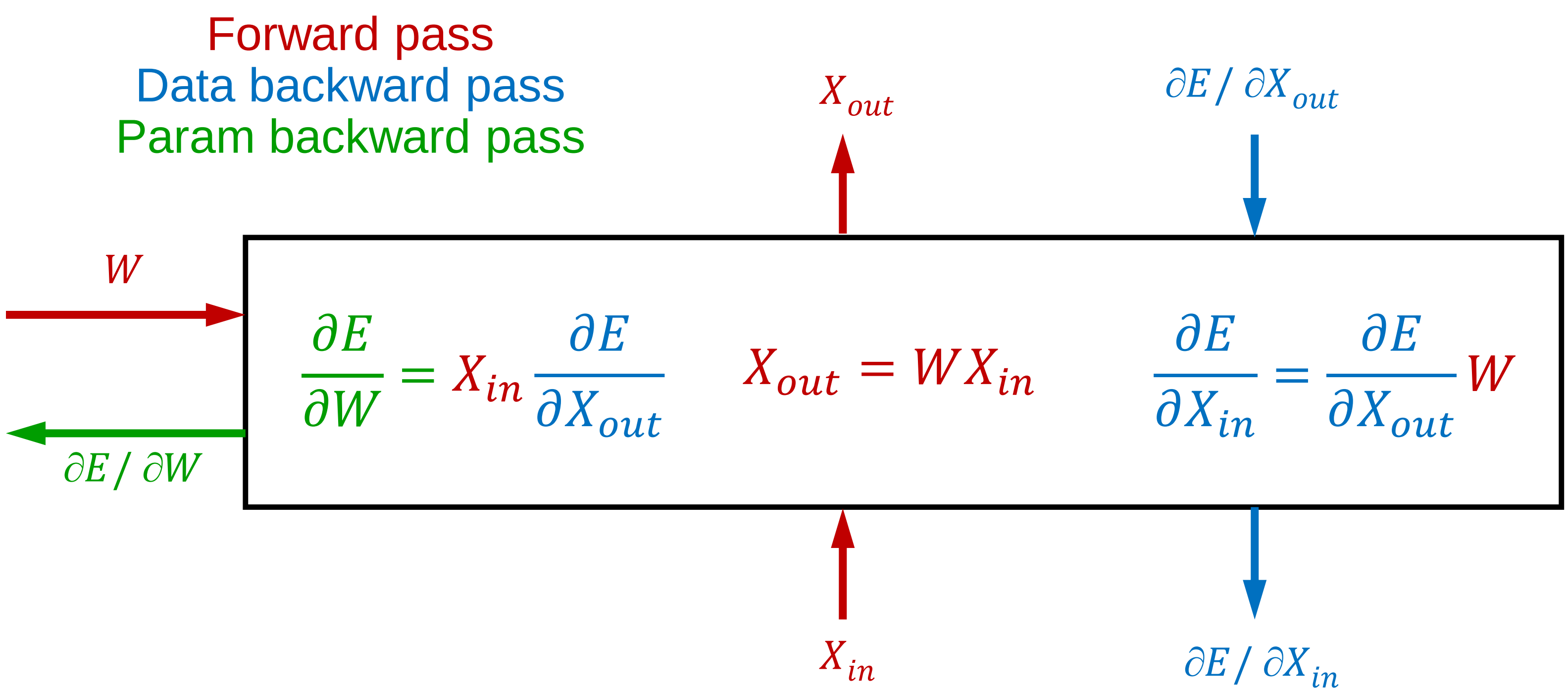
Gradient back-propagation rule:

The gradient relative to the input (either W or X_{in}) is equal to the gradient relative to the output (X_{out}) times the Jacobian of the transfer function (respectively $\frac{\partial X_{out}}{\partial W}$ or $\frac{\partial X_{out}}{\partial X_{in}}$, left vector multiplication)

$$\frac{\partial F(W, X_{in})}{\partial X_{in}} \equiv \frac{\partial X_{out}}{\partial X_{in}} \qquad \frac{\partial E}{\partial X_{in}} = \frac{\partial E}{\partial X_{out}} \frac{\partial X_{out}}{\partial X_{in}}$$

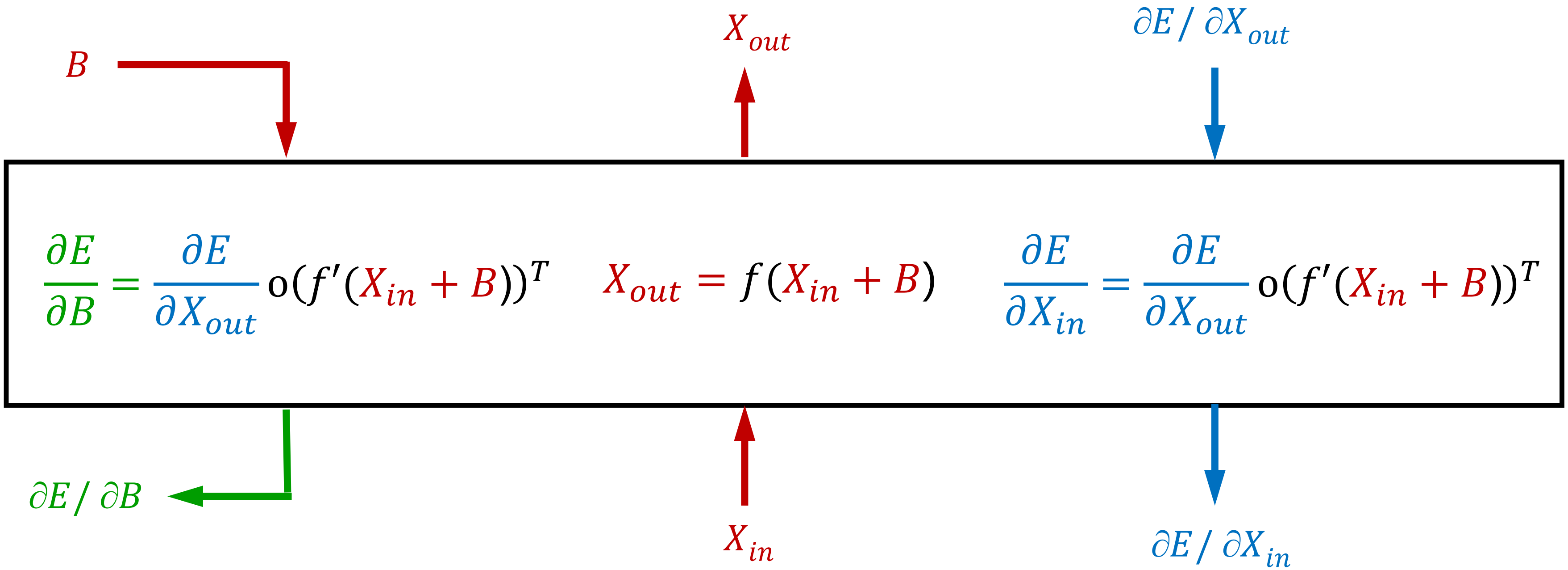
$$\frac{\partial F(W, X_{in})}{\partial W} \equiv \frac{\partial X_{out}}{\partial W} \qquad \frac{\partial E}{\partial W} = \frac{\partial E}{\partial X_{out}} \frac{\partial X_{out}}{\partial W}$$

Linear module (adapted from Yann LeCun)



Note: X_{in} and X_{out} are regular (column) vectors and W is a matrix while $\partial E / \partial X_{in}$ and $\partial E / \partial X_{out}$ are transpose (row) vectors, this is because $\mathrm{d}E = (\partial E / \partial X).\mathrm{d}X$. $\partial E / \partial W$ is a transposed matrix which is the *outer* product of the regular and transpose vectors X_{in} and $\partial E / \partial X_{out}$.

Pointwise module (adapted from Yann LeCun)



Notes: B is a bias vector on the input. X_{in}, X_{out} and B are regular (column) vectors all of the same size while $\partial E / \partial X_{in}$ and $\partial E / \partial X_{out}$ and $\partial E / \partial B$ are transpose vectors also of the same size. f is a scalar function applied pointwise on $X_{in} + B$. f' is the derivative of f and is also applied pointwise. The multiplication by $(f'(X_{in} + B))^T$ is also performed pointwise (Hadamard product denoted “o” here).

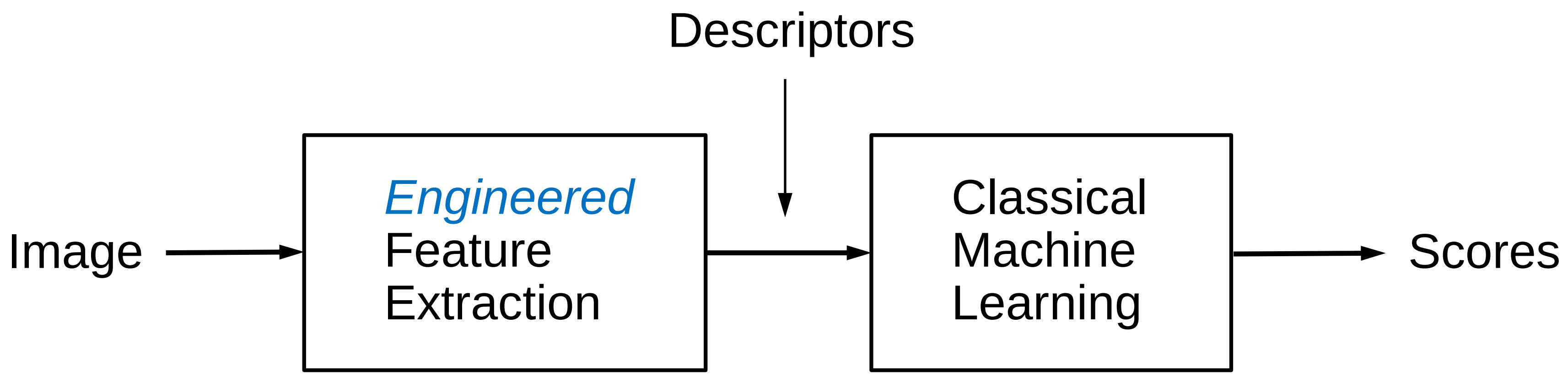
Non-linear functions

- Sigmoid: $z = \frac{1}{1+e^{y}}$
- Hyperbolic tangent: $z = \tanh y$
- Rectified Linear Unit (ReLU): $z = \max(0, y)$
- Programmable ReLU (PReLU) : $z = \max(\alpha y, y)$ with α learned (i.e. $\alpha \subset W$)
- …
- Appropriate non-linear functions leads to better performance and/or faster convergence
- Avoid vanishing / exploding gradients

Neural Networks in practice

- Good news is that `autograd` automatically and transparently takes care of gradients computation and propagation; you just have to call `.backward()`
- You only have to define the forward network sequence
- You still have to select various hyper-parameters and to organize:
  - iterations
  - batch processing
  - learning rate schedule
  - possibly data augmentation

Classical Image classification

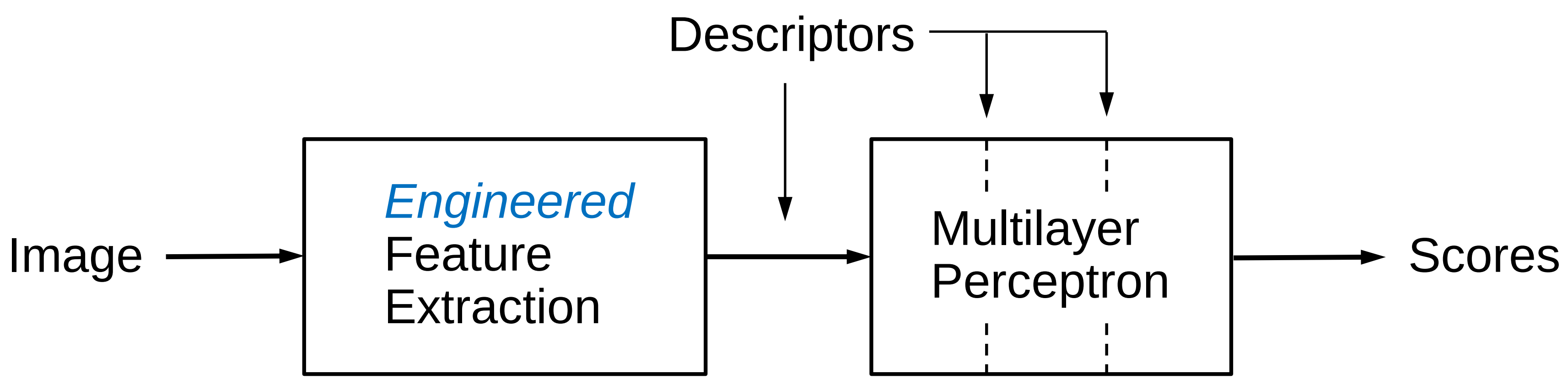


Color Histograms
Gabor Transforms
Bags of SIFTs
Fisher Vectors
…

Support Vector Machines
Multilayer Perceptrons
Random Forests
…

Plus: multiple features, early or late fusion, re-scoring …

Classical Image classification

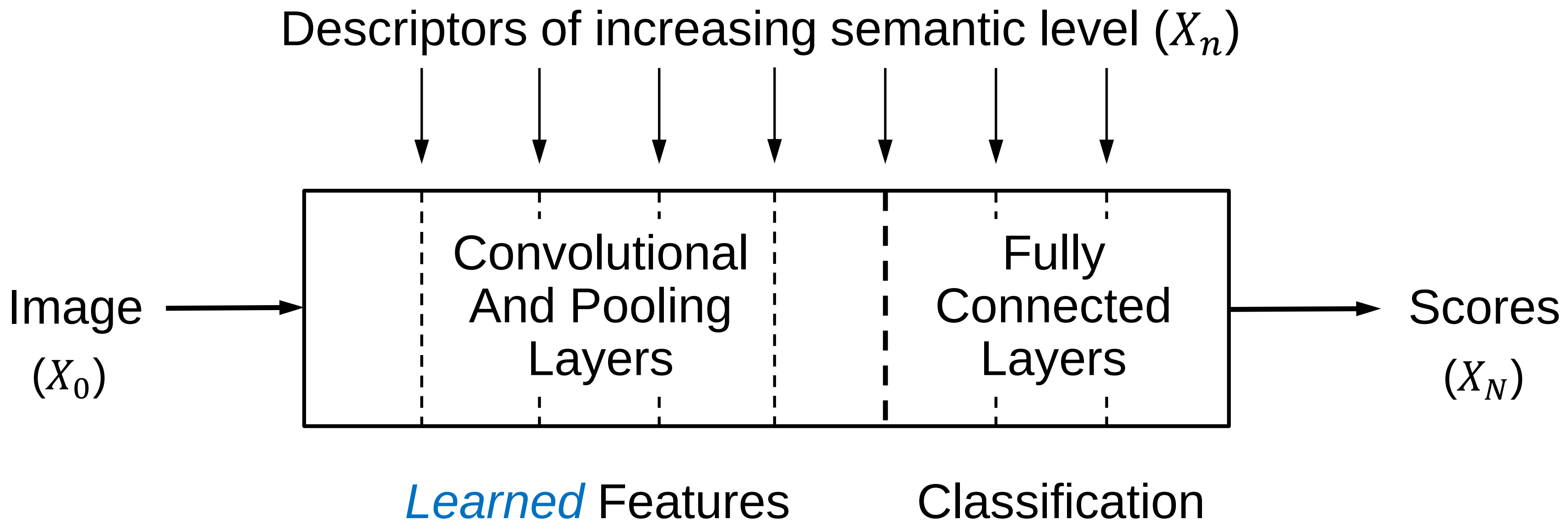


Color Histograms
Gabor Transforms
Bags of SIFTs
Fisher Vectors
…

Typically 3 layers
Not really better than SVMs or Random Forests

Still classical since 3-layer MLPs are at least 30 years old

Deep "end-to-end" Image classification

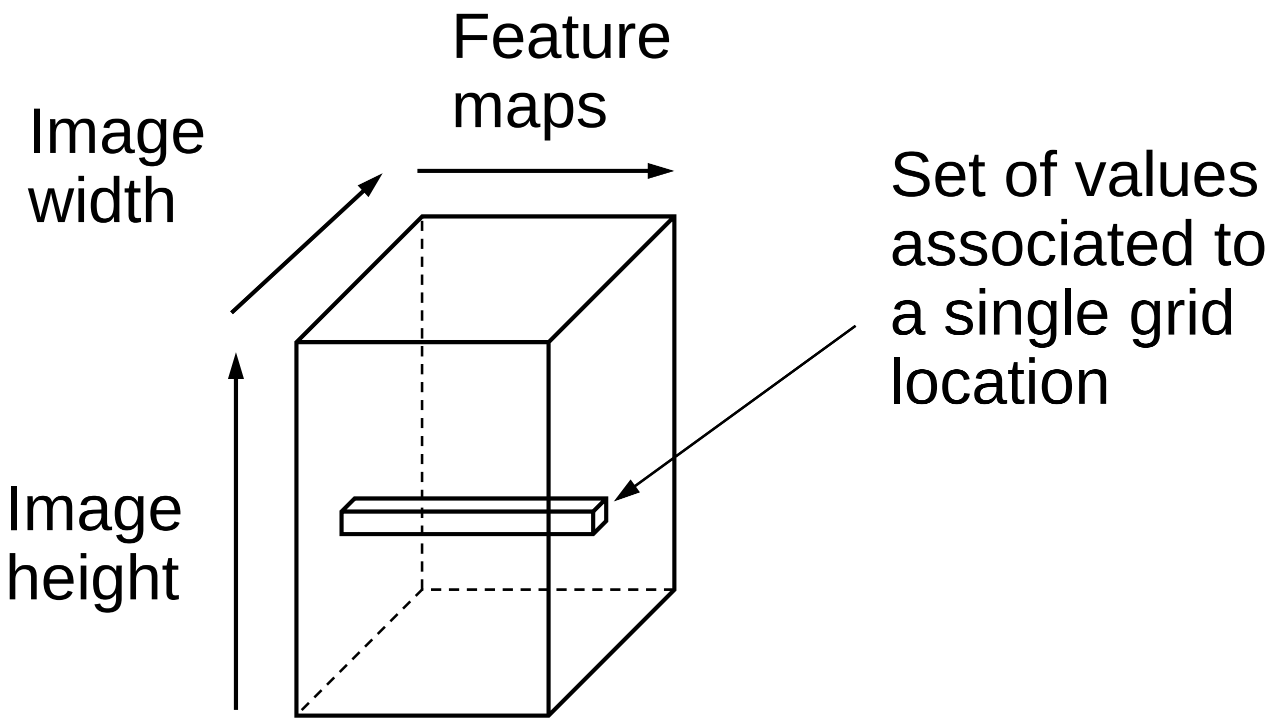


- Fuzzy boundary between feature extraction and classification even if there is a transition between convolutional and fully connected layers
- *End-to-end learning*: features (descriptors) themselves are learned (by gradient descent) too, not engineered
- Possible only via the use of *convolutional* layers

Convolutional layers (2D grid case)

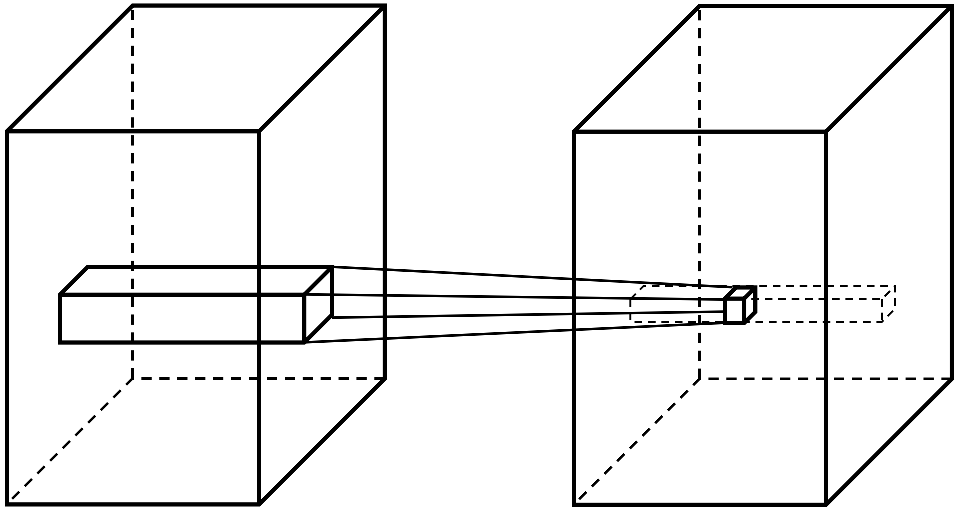
- Alternative to the “all to all”(vector to vector) connections
- Preserves the 2D image topology via “feature maps”
- X_n are 3D data (“tensors”) instead of vectors
- 2 of the dimensions are aligned with the image grid
- The third dimension is a set of values associated to a grid location (gathered in a vector per location but without associated topology)
- Each component in the third dimension correspond to a “map” aligned with the image grid
- Each data tensor is a “stack” of features maps
- Translation-invariant (relatively to the grid) processing

3D tensor data (2D grid case)



Input image data is a special case with 3 feature maps corresponding to the RGB planes and sometimes 4 or even more for RGB-D or for hyper-spectral (satellite) image data.

Convolutional layers (2D grid case)



- Each map point is connected to all maps points of a fixed size neighborhood in the previous layer
- Weights between maps are shared so that they are invariant by translation

Convolutional layers (2D grid case)

- Combination of:
  - convolutions within the image plane
  - “all to all” within the map dimension
- Separable or non-separable combinations
- Resolution changes across layers: stride and pooling
- Example: AlexNet

Classical image convolution (2D to 2D)

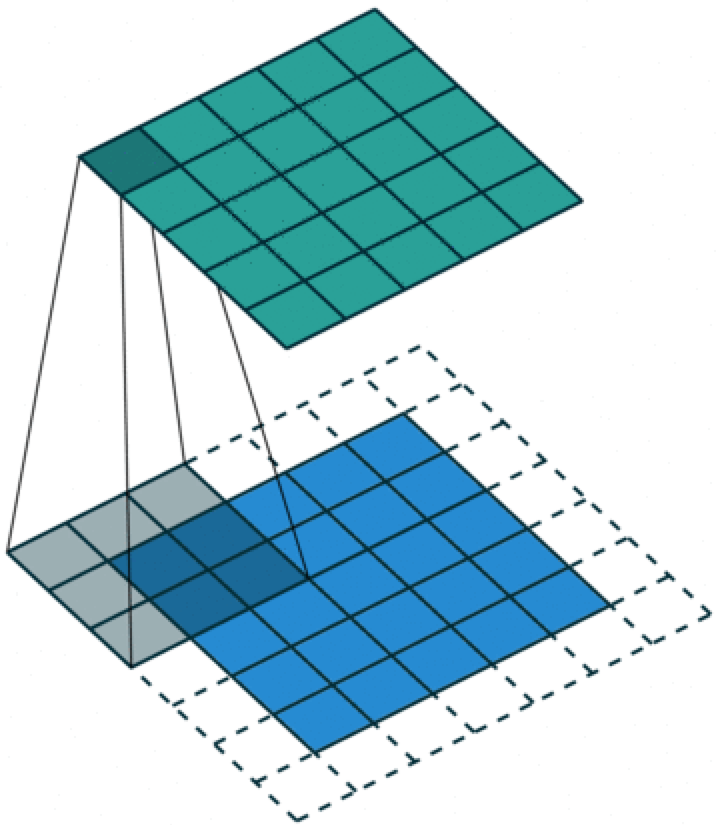
- Classical image convolution (2D to 2D):

$$O(i,j) = (K * I)(i,j) = \sum_{(m,n)} K(m,n) I(i-m, j-n)$$

- Convolutional layer (3D to 3D):
- m and n : within a window around the current location, corresponding to the filter size
- $K(m,n)$: convolution kernel
- Example: (circular) Gabor filter:

$$K(m,n) = \frac{1}{2\pi\sigma^2} . e^{-\frac{m^2+n^2}{2\sigma^2}} . e^{2\pi i \frac{m.\cos\theta + n.\sin\theta}{\lambda}}$$

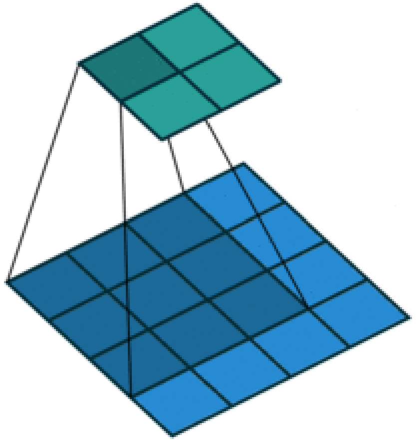
Classical image convolution (2D to 2D)



3x3 convolution, no stride, half padding

Animation from https://github.com/vdumoulin/conv_arithmetic/

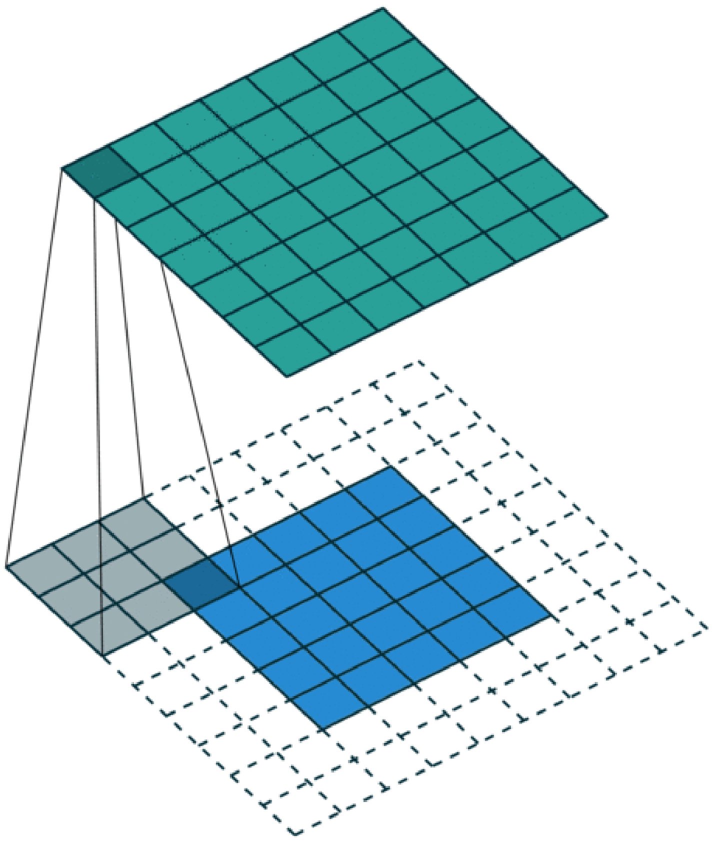
Classical image convolution (2D to 2D)



3×3 convolution, no stride, no padding

Animation from https://github.com/vdumoulin/conv_arithmetic/

Classical image convolution (2D to 2D)



3×3 convolution, no stride, full padding

Animation from https://github.com/vdumoulin/conv_arithmetic/

Convolutional layers

- Set of image convolution (2D to 3D):

$$O(l,i,j) = (K(l) * I)(i,j) = \sum_{(m,n)} K(l,m,n) I(i-m, j-n)$$

- Convolutional layer: multiple maps (planes) both in input and output (3D to 3D, plus bias):

$$O(l,i,j) = B(l) + \sum_{(k,m,n)} K(k,l,m,n) I(k, i-m, j-n)$$

- k and l: indices of the feature maps in the input and output layers

- m and n: within a window around the current location, corresponding to the feature size

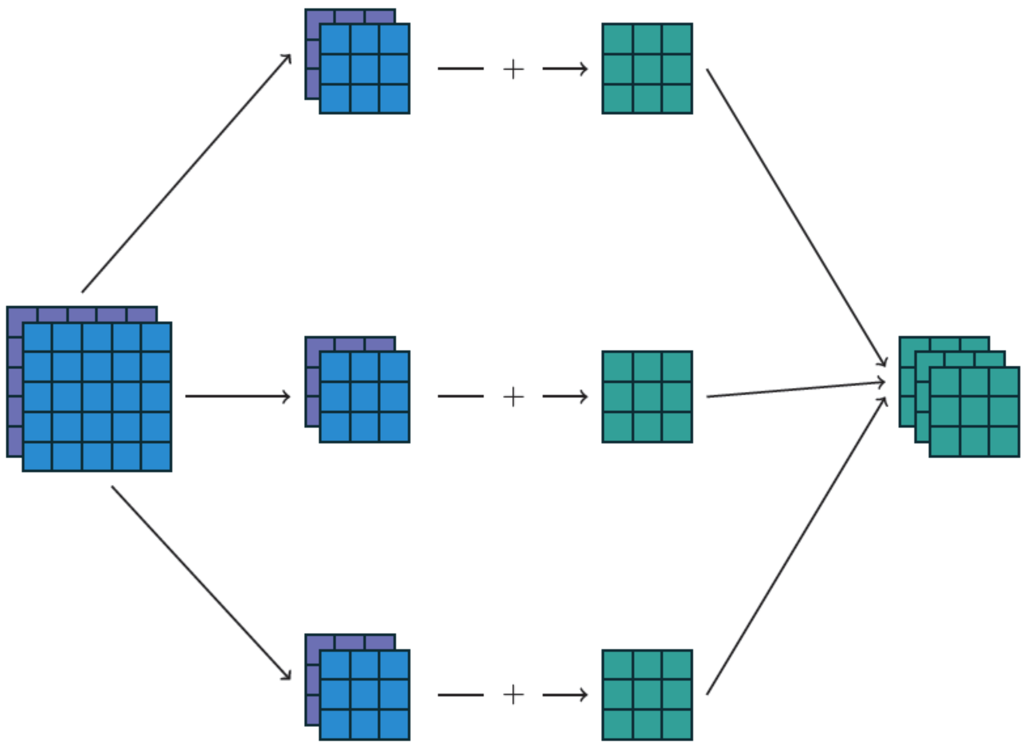
Convolutional layers

- Convolutional layer: multiple maps (planes) both in input and output (3D to 3D, plus bias):

$$O(l,i,j) = B(l) + \sum_{(k,m,n)} K(k,l,m,n) I(k,i-m,j-n)$$

- Operation relative to (m,n) : convolution
- Operation relative to (k,l) : matrix multiplication plus bias (equals affine transform)
- Combination of:
  – Convolution within the image plane, image topology
  – Classical all to all “perpendicularly” to the image plane, no topology
- If image size and filter size = 1: fully connected “all to all”

Convolutional layers (3D to 3D)



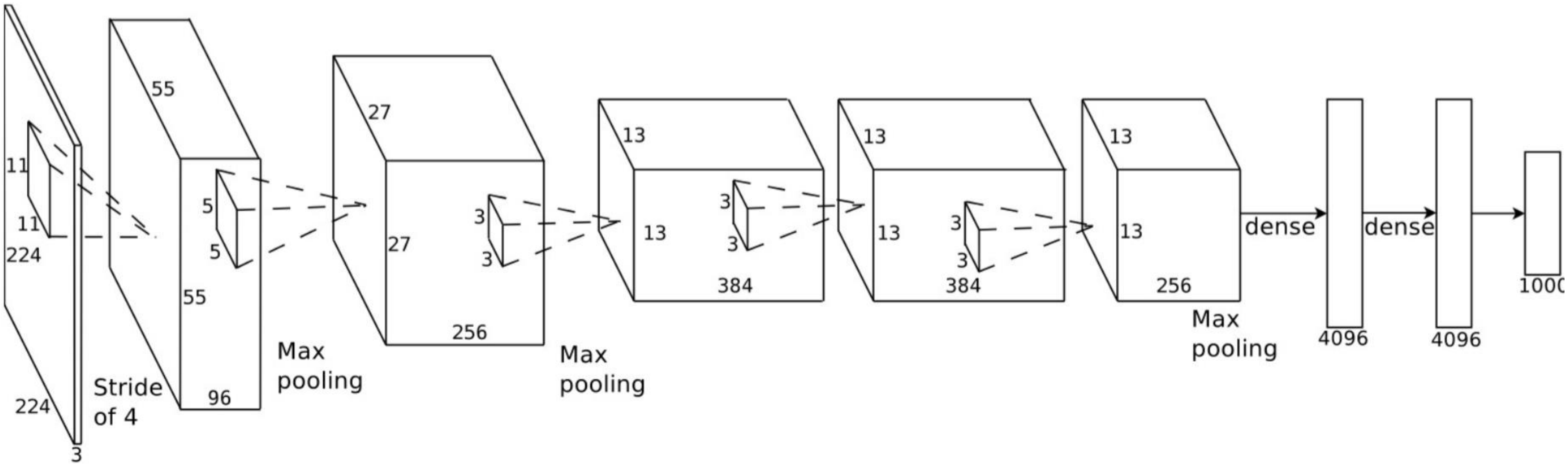
2(input)×3×3×3(output) convolution, no stride, no padding

Illustration from https://arxiv.org/abs/1603.07285

AlexNet (ImageNet Challenge 2012)

[Krizhevsky et al., 2012]

- 7 hidden layers, 650K units, 60M parameters (W)
- GPU implementation (50× speed-up over CPU)
- Trained on two GTX580-3GB GPUs for a week

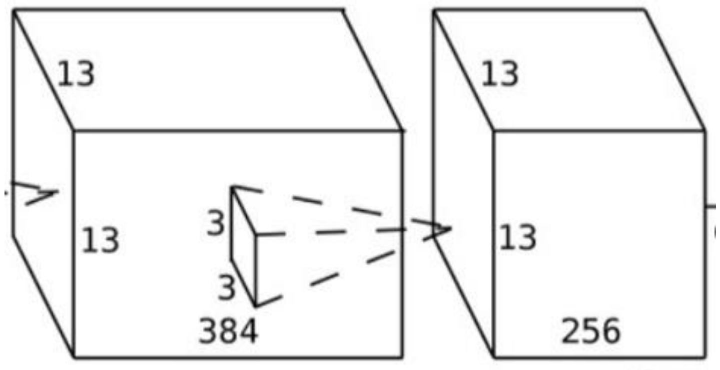


A. Krizhevsky, I. Sutskever, and G. Hinton, ImageNet Classification with Deep Convolutional Neural Networks, NIPS 2012

Convolutional layers

- The convolution layer kernel is: $(D+2)$-dimensional for D-dimensional input data, e.g. $D=2$ for still images, $D=3$ for video segments or scanner images.
- For color images, the RGB (or YUV or HSV …) planes directly enter the first layer as a 3D volume of size width × height × 3
- There is one unit (neuron) per “pixel” in the output D-dimensional topology and per output feature map
- Unit set: set of units associated to a D-dimensional grid location, one unit per output feature map, one set per grid location
- There is a single translation-invariant $(D+2)$-dimensional kernel per layer for mapping input pixel vectors to output pixel vectors at all D-dimensional grid locations

AlexNet “conv5” example



- Number of units (“neurons”) in a layer (= size of the output tensor): output image width (13) × output image height (13) × number of output planes (256) = 43,264
- Number of weights in a layer (= number of weights in a layer): number of input planes (384) × number of output planes (256) × filter width (3) × filter height (3) = 884,736 (884,992 including biases)
- Number of connections: number of grid locations × number of weights in a unit set (excluding biases) = 149,520,384

Resolution changes and side effects

- Side (border) effect:
  - crop the output “image” relative to the input one and/or
  - pad the image if the filter expand outside
- Resolution change (generally reduction):
  - Stride: subsample, e.g. compute only one out of N, and/or
  - Pool: compute all and apply an associative operator to compute a single value for the low resolution location from the high resolution ones
- Common pooling operators: maximum or average
- Pooling correspond to a separate back-propagation module (as the linear and non-linear parts of a layer)

Dropout

- Regularization technique
- During training, at each epoch, neutralize a given (typically 0.2 to 0.5) proportion of randomly selected connections
- During prediction, keep all of them with a multiplicative compensating factor
- Avoid concentration of the activation on particular connections
- Much more robust operation
- Faster training, better performance

Softmax

- Normalization of output as probabilities (positive values summing to 1) for the multi-class problem (i.e. target categories are mutually exclusive)
- $z_i = \frac{e^{y_i}}{\sum_j e^{y_j}}$
- Not suited for the multi-label case (i.e. target categories are not mutually exclusive)
- Associated loss function is cross-entropy

Cross-entropy loss (multi-class)

- p_i : probability vector for class i
- l_i : truth value for class i ("one hot encoding")
- $L = \sum_i -(l_i \log p_i)$
- For exclusive classes, l_i is equal to 1 only for the right class i_0 and to 0 otherwise:
- $L = -\log p_{i_0}$ $\quad$ ($\log 1 = 0$ and $\log 0 = -\infty$)
- Forces p_{i_0} to be close to 1, very high loss value if p_{i_0} is close to 0 $\rightarrow$ faster convergence
- Other p_i indirectly forced to be close to 0 because the p_is sums to 1
- With softmax: forces y_{i_0} to be greater than the other y_is

Cross-entropy loss (multi-label)

- Non-exclusive categories are called labels and are seen as independent, each with two-classes
- p_i : probability vector for label i
- l_i : truth value for label i (either 0 or 1)
- Sigmoid “normalization”: $p_i = \frac{1}{1+e^{-y_i}}$ and $1 - p_i = \frac{1}{1+e^{y_i}}$
- $L = \sum_i -(l_i \log p_i + (1 - l_i)\log(1 - p_i))$
- Same formula as for multi-class with a two-class problem for each label
- Sum of CE Losses per label
- Note: works also if l_i has non-binary values (probabilities of the true distribution)

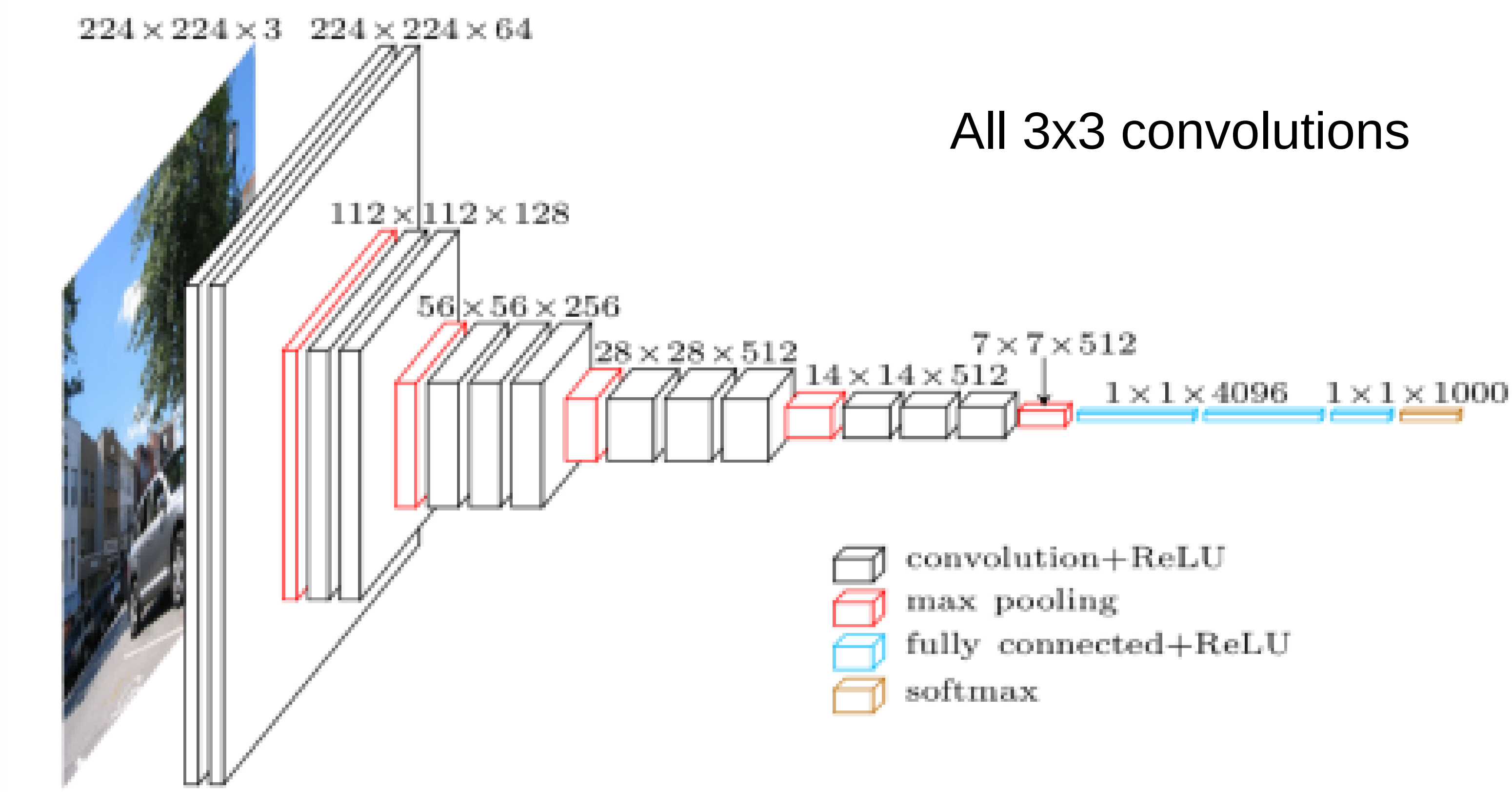
Yann LeCun recommendations

- Use ReLU non-linearities (tanh and logistic are falling out of favor)
- Use cross-entropy loss for classification
- Use Stochastic Gradient Descent on minibatches
- Shuffle the training samples
- Normalize the input variables (zero mean, unit variance)
- Schedule to decrease the learning rate
- Use a bit of L1 or L2 regularization on the weights (or a combination)
  - But it's best to turn it on after a couple of epochs
- Use “dropout” for regularization
  - Hinton et al 2012 http://arxiv.org/abs/1207.0580
- Lots more in [LeCun et al. “Efficient Backprop” 1998]
- Lots, lots more in “Neural Networks, Tricks of the Trade” (2012 edition) edited by G. Montavon, G. B. Orr, and K-R Müller (Springer)

“Recent” trends

- VGG and GoogLeNet (16-19 and 22 layers)
- Residual networks (152 layers with “shortcuts”)
- Stochastic depth networks (up to 1202 layers)
- Dense Networks
- Weakly supervised / unsupervised learning
- Generative adversarial networks
- Segmentation networks
- Multimodal embeddings

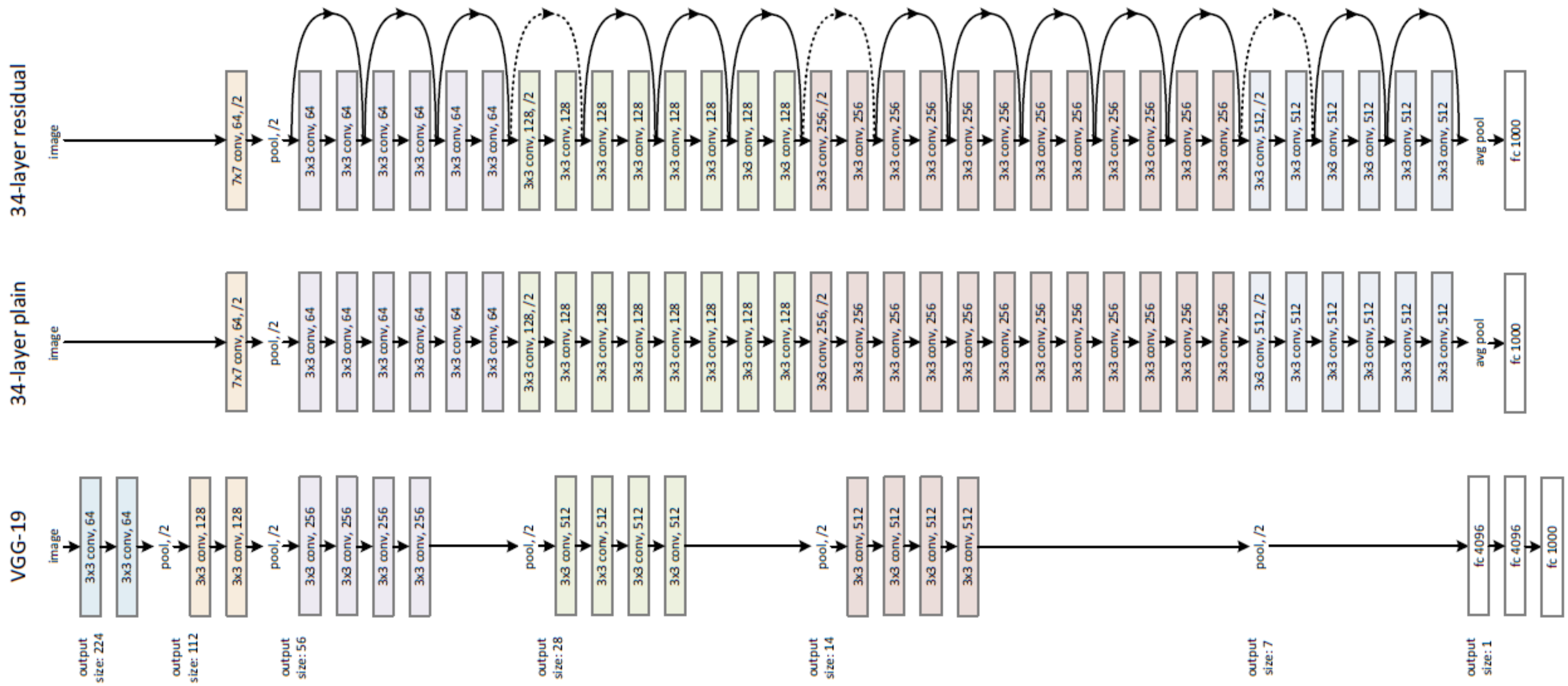
VGG Network (very deep)



All 3x3 convolutions

Simonyan and Zisserman, Andrew: *Very Deep Convolutional Networks for Large-Scale Image Recognition*, CVPR 2014.

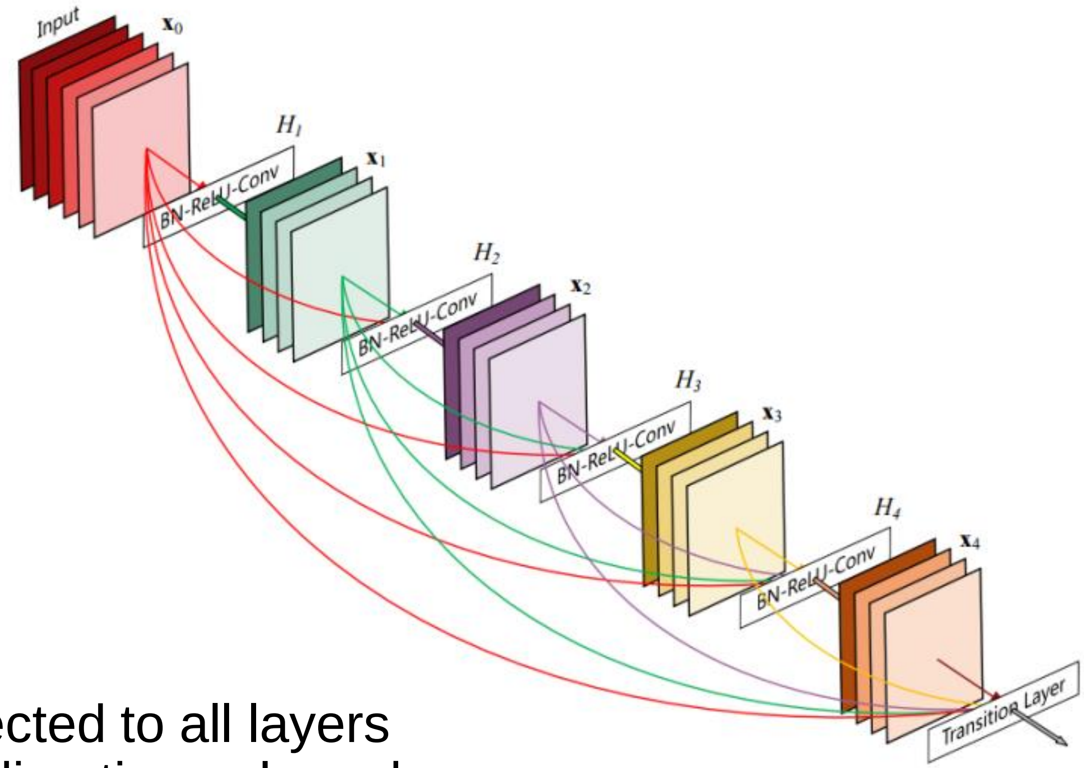
Residual networks (ultra deep)



Ultra deep network with “shortcuts”

He, Zhang, Ren and Sun: Deep Residual Learning for Image Recognition, CVPR 2015

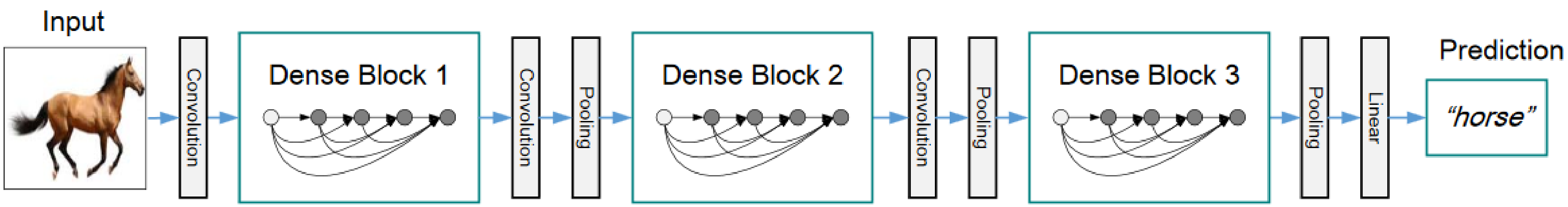
Dense networks



All layers connected to all layers
(in the forward direction only and
without resolution change

Huang et al.: *Densely Connected Convolutional Networks*, CVPR 2016

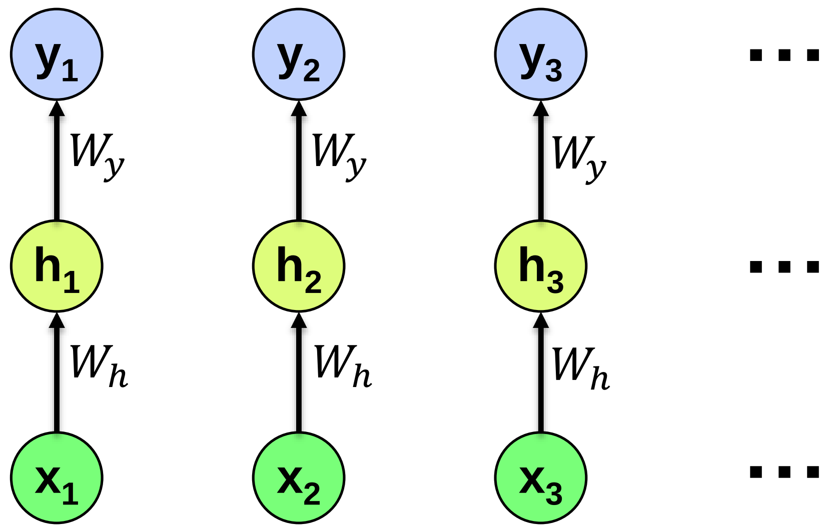
Dense networks



A deep DenseNet with three dense blocks
The layers between blocks are transition layers that change the resolution via convolution and pooling

Huang et al.: *Densely Connected Convolutional Networks*, CVPR 2016

Feed-forward network on a sequence



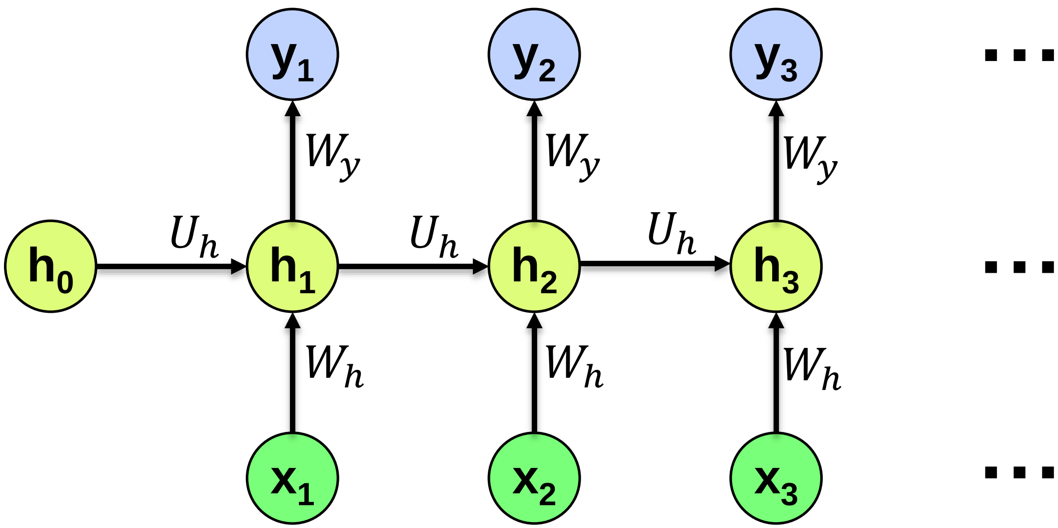
$$h_t = \sigma(W_h x_t + b_h)$$
$$y_t = \sigma(W_y h_t + b_y)$$

Independent predictions: no history

Training on samples

Only two linear layers here: not deep (usually more)

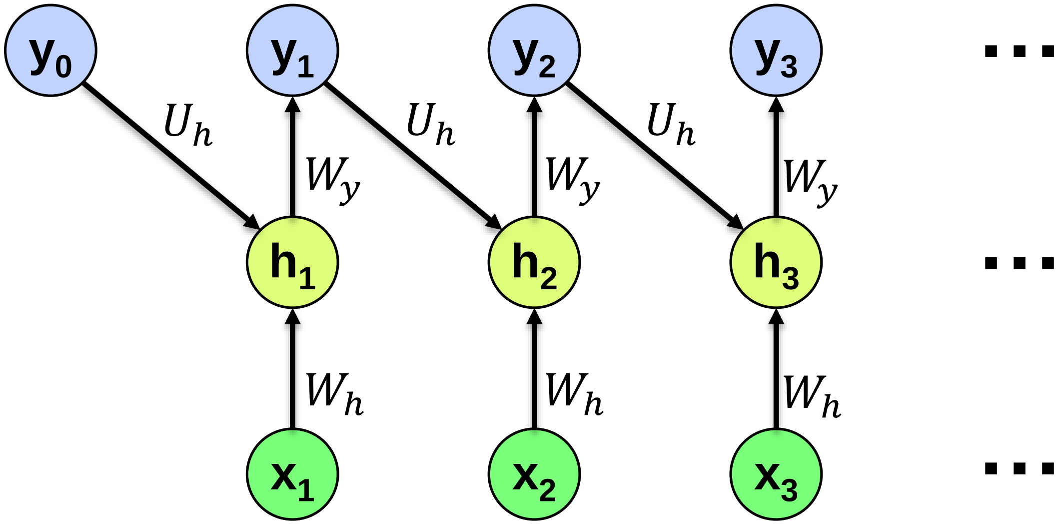
Simple recurrent network (Elman)



$$h_t = \sigma(W_h x_t + U_h h_{t-1} + b_h)$$
$$y_t = \sigma\left(W_y h_t + b_y\right)$$

Sequence (past) history is represented in the hidden states
Training on sequences (unfolded loop)
Back-propagation through many hidden states: deep

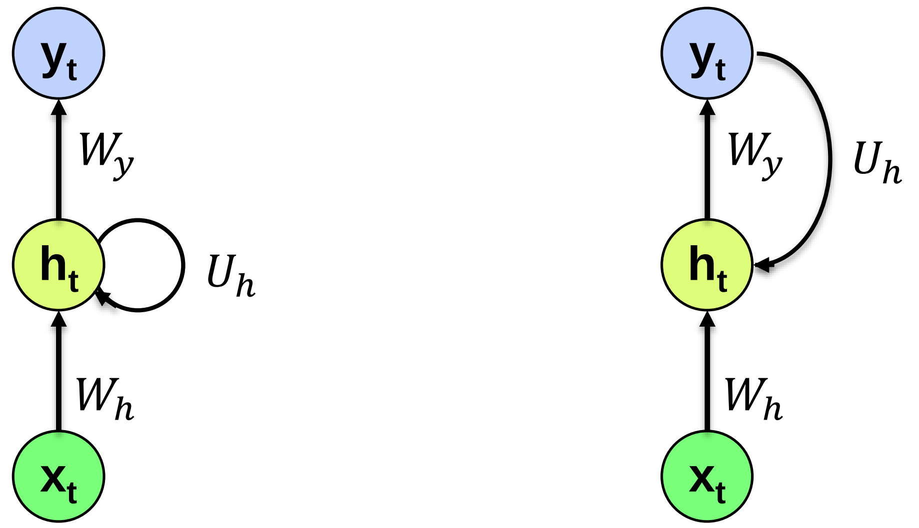
Simple recurrent network (Jordan)



$$h_t = \sigma(W_h x_t + U_h y_{t-1} + b_h)$$
$$y_t = \sigma\left(W_y h_t + b_y\right)$$

Sequence (past) history is represented in the hidden states
Training on sequences (unfolded loop)
Back-propagation through many hidden states: deep

Folded (usual) representations



Elman

Jordan

Recurrent neural networks

- Perform sequence-to-sequence transformations
- Learns patterns in sequences
- Used in speech and in natural language processing
- Used in video processing (action recognition)
- Simple RNNs have limitations (unstable gradients)
- Variants with “memory cells”:
  - Long Short-Term Memory (Hochreiter and Schmidhuber, 1997)
  - Gated Recurrent Units (Cho et al., 2014) (simplified LSTM)
  - Avoid exploding or vanishing gradients on long sequences
  - Can “count”

Word embeddings

- Map words in a D-dimensional space with semantic distances and relations roughly preserved

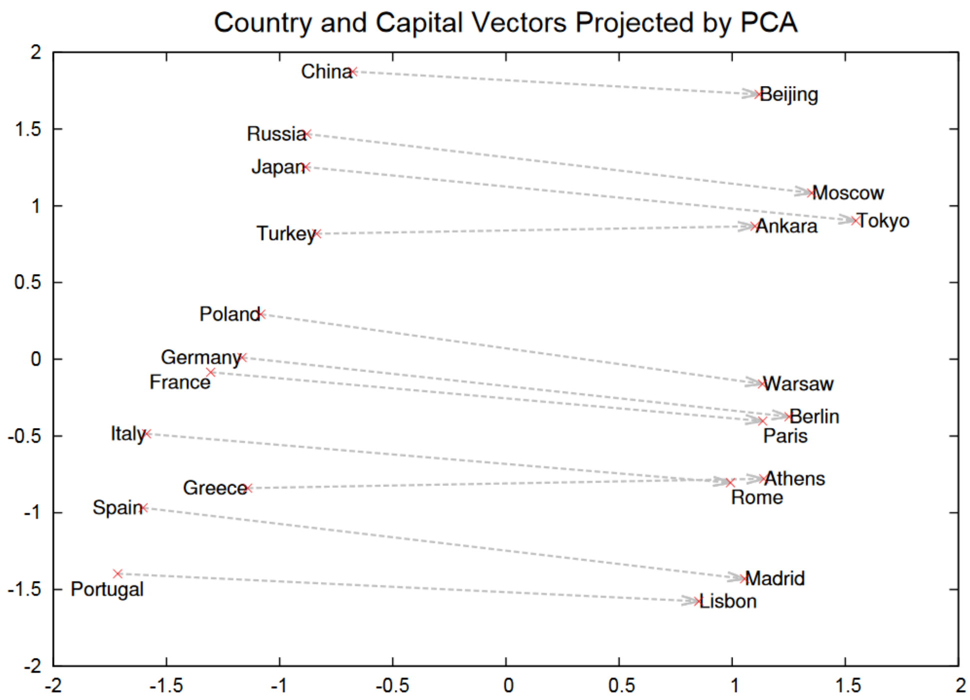


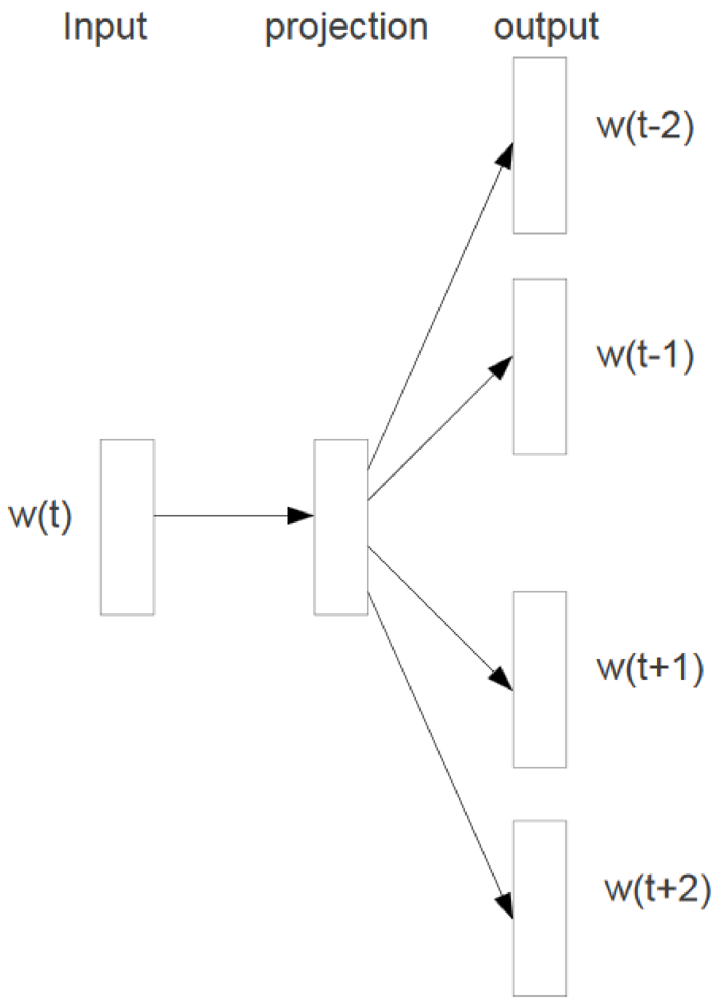
Figure 2: Two-dimensional PCA projection of the 1000-dimensional Skip-gram vectors of countries and their capital cities. The figure illustrates ability of the model to automatically organize concepts and learn implicitly the relationships between them, as during the training we did not provide any supervised information about what a capital city means.

From Mikolov, 2013 (Word2Vec)

Word2Vec (Mikolov et al., 2013)

- Words are represented by “1-hot encoding”
- Encoder-decoder architectures
  - Encoder: V dims to D dims linear map(s)
  - Decoder: D dims to V dims linear map(s)
  - V: vocabulary size, D: embedding size
- Two variants:
  - CBOW: predict single words from their neighbors
  - Skip-gram: predict neighbors from single words (better)
- The intermediate representation is the embedding
- Unsupervised learning: from huge amounts of raw data
- Learning by gradient descent

Word2Vec skip grams



1 encoding matrix
4 decoding matrices

All source and target vectors are 1-hot encoded